UNIT 1

HEAT CONDUCTION

OBJECTIVES:

1. To learn different modes of heat transfer
2. To provide knowledge heat transfer through conduction, convection & radiation

Outcomes:

1. Student will be able to understand how heat & energy is transferred between elements of a system.
2. Able to solve problems involving one or more modes of heat transfer.

INTRODUCTION

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Thermodynamics and heat transfer

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Introduction

- Heat is fundamentally transported, or “moved,” by a temperature gradient; it flows or is transferred from a high temperature region to a low temperature one. An understanding of this process and its different mechanisms are required to connect principles of thermodynamics and fluid flow with those of heat transfer.

Thermodynamics and Heat Transfer

- Thermodynamics is concerned with the amount of heat transfer as a system undergoes a process from one equilibrium state to another, and it gives no indication about how long the process will take. A thermodynamic analysis simply tells us how much heat must be transferred to realize a specified change of state to satisfy the conservation of energy principle.
- In practice we are more concerned about the rate of heat transfer (heat transfer per unit time) than we are with the amount of it. For example, we can determine the amount of heat transferred from a thermos bottle as the hot coffee inside cools from 90°C to 80°C by a thermodynamic analysis alone.
- But a typical user or designer of a thermos is primarily interested in how long it will be before the hot coffee inside cools to 80°C, and a thermodynamic analysis cannot answer this question. Determining the rates of heat transfer to or from a system and thus the times of cooling or heating, as well as the variation of the temperature, is the subject of heat transfer (Figure 1.1).

Fig. 1.1 Heat transfer from the thermos

- Thermodynamics deals with equilibrium states and changes from one equilibrium state to another. Heat transfer, on the other hand, deals with systems that lack thermal equilibrium, and thus it is a nonequilibrium phenomenon. Therefore, thestudy of heat transfer cannot be based on the principles of thermodynamics alone.
- However, the laws of thermodynamics lay the framework for the science of heat transfer. The first law requires that the rate of energy transfer into a system be equal

to the rate of increase of the energy of that system. The second law requires that heat be transferred in the direction of decreasing temperature (Figure 1.2).

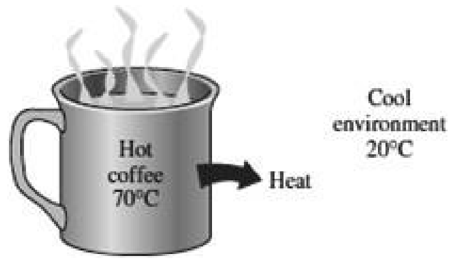

Fig. 1.2 Heat transfer from high temperature to low temperature

Application Areas of Heat Transfer

- Many ordinary household appliances are designed, in whole or in part, by using the principles of heat transfer. Some examples:
- Design of the heating and air-conditioning system, the refrigerator and freezer, the water heater, the iron, and even the computer, the TV, and the VCR
- Energy-efficient homes are designed on the basis of minimizing heat loss in winter and heat gain in summer.
- Heat transfer plays a major role in the design of many other devices, such as car radiators, solar collectors, various components of power plants, and even spacecraft.
- The optimal insulation thickness in the walls and roofs of the houses, on hot water or steam pipes, or on water heaters is again determined on the basis of a heat transfer analysis with economic consideration (Figure 1.3)

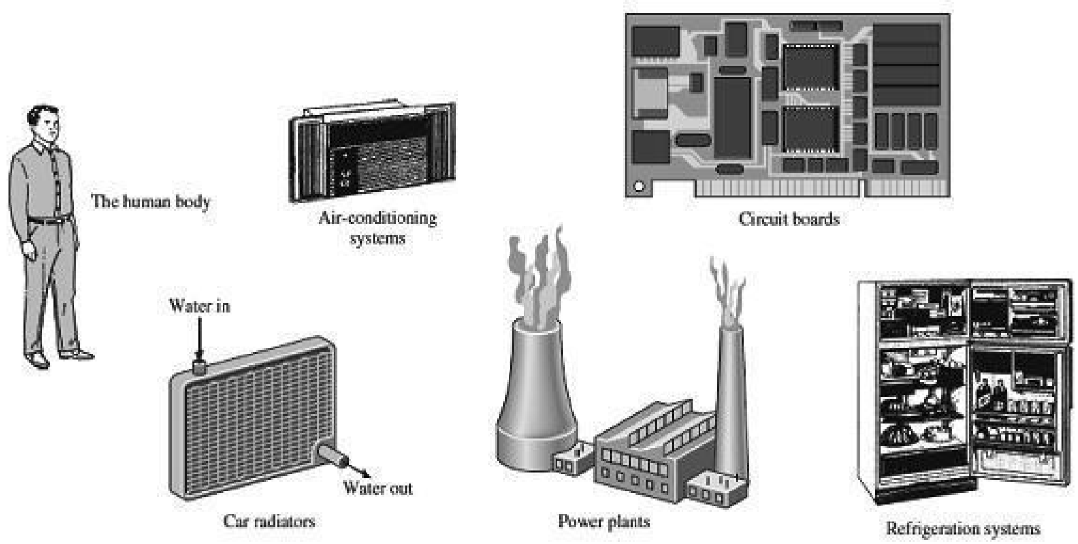

Fig. 1.3 Application of heat transfer

MODES OF HEAT TRANSFER

Heat transfer is defined as the transfer of heat from one region to another by virtue of the temperature difference between them. The devices for transfer of heat are called heat exchangers. The concept of heat transfer is necessary for designing heat exchangers like boilers, evaporators, condensers, heaters and many other cooling and heating systems.

There are three modes of Heat transfer as follows:

1. Conduction
2. Convection
3. Radiation.

1. Conduction

Heat is always transferred by conduction from high temperature region to low temperature region. The conduction heat transfer is due

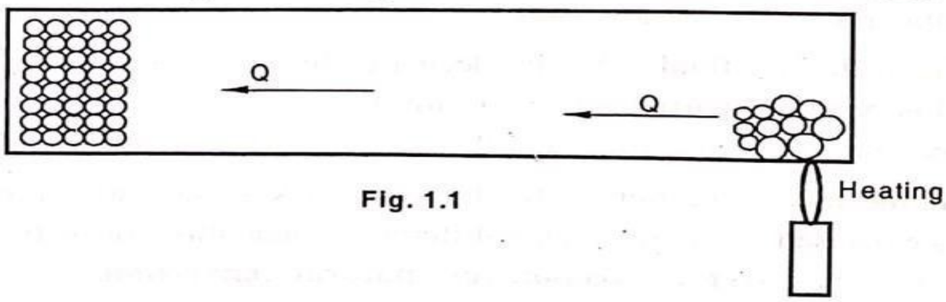

Fig. 1.1

to the property of matter and molecular transport of heat between two regions due to temperature difference.

When one end of a rod gets heated, the atoms in that end get enlarged and vibrated due to heating. The enlarged, vibrated atoms touch the adjacent atom and heat is transferred. Similarly, all the atoms are heated, thereby the heat is transferred to the other end. This type of heat transfer is called as conduction heat transfer.

In solids, heat is conducted by

1. Atomic vibration – The faster moving, vibrating atoms in the hot area transfer heat to the adjacent atoms.
2. By transport of free electrons.

Heat is also conducted in liquid and gases by the following mechanism.

1. The kinetic energy (K.E) of a molecule is a function of temperature. When these molecules' temperature increases, the K.E. increases.

2. The molecule from the high temperature region collides with a molecule from the low temperature region and thus heat is transferred.

CONDUCTION

Most of the heat transfer problems involve a combination of all the three modes of heat transfer. But it will be useful, if we study each mode of heat transfer one by one. Hence, in the forth coming section, we can study conduction, convection and radiation separately and in some cases we can study with combination.

1.2.1 Fourier's law of heat conduction

Fourier's law states that the Conduction heat transfer through a solid is **directly proportional to**

1. The area of section (A) at right angle to the direction of heat flow.
2. The change in temperature (dT) in between the two faces of the slab and
3. **Indirectly proportional to** the thickness of the slab (dx).

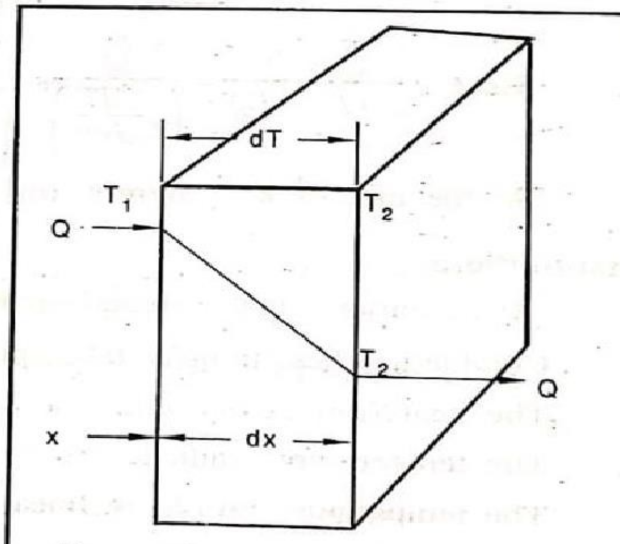

Fig. 1.2

$$Q \propto A \frac{dT}{dx}$$

where Q = heat conducted in (Watts) W.

A = surface area of heat flow in m^2. (perpendicular to the direction of heat flow)

dT = temperature difference between the faces of the slab in °C or K

dx = thickness of the slab in m.

$$\text{So, } Q = -kA \frac{dT}{dx}$$

Here dT is negative. Because $dT = T_2 - T_1$. (Change in temp.) Since T_2 is less than T_1, dT is negative.

So we get the equation

$$Q = -kA\frac{(T_2 - T_1)}{dx} = kA\frac{(T_1 - T_2)}{dx}$$

Here $k =$ Constant of proportionality and is called **therma conductivity of the material.**

$$Q = kA\frac{(T_1 - T_2)}{dx}$$

$$\text{So, } k = \frac{Q \times dx}{A\,(T_1 - T_2)} = \frac{Q}{\left(\frac{A\,dT}{dx}\right)} = \frac{W}{\left(\frac{m^2 \times {}^\circ C}{m}\right)} = \text{W/m}^\circ\text{C}$$

So the unit of k is W/m°C (or) W/mK

1.3 THERMAL CONDUCTIVITY (*k*)

Thermal conductivity, k of a material is defined as the heat conducted through a body of unit area and unit thickness in unit time with unit temperature difference.

1.3.1 Thermal conductivity of solids:

Solids are classified into 4 types

- Metalic solids (Pure metals)
- Alloys
- Non-Metalic solids
- Powder and Porous solids

Thermal Resistance

The heat transfer process is analogous to the flow of electricity. According to Ohm's law,

$$\text{Current } (I) = \frac{\text{Voltage difference } (dV)}{\text{Electrical resistance } (R)}$$

$$Q = \frac{kA\,(T_1 - T_2)}{L}$$

It can be rewritten as

$$Q = \frac{T_1 - T_2}{L/kA}$$

where $\frac{L}{kA}$ is called thermal resistance R_{th}.

$$\text{So, } R_{th} = \frac{L}{kA}$$

The reciprocal of thermal resistance is thermal conductance.

$$\text{So, } Q = \frac{\Delta T}{R}$$

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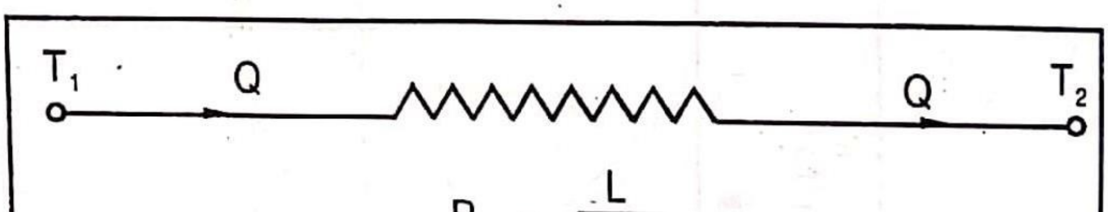


Fig. 1.8(a)

R for different figures are given in page No: 44, 45, 46 and 47 of HMT data book by CPK.

2. Convection

The heat transfer between a surface and the surrounding fluid which are at different temperatures, is called convection heat transfer. Convection heat transfer is defined as a process of heat transfer by the combined action of **heat conduction** and **mixing motion**.

Consider a container full of water. Heat is conducted through container wall.

(i) First of all, heat is transferred from hot surface of wall to adjacent fluid purely by **conduction**.

(ii) Then, the hot fluid's density decreases by increase in temperature. This hot fluid particles move to top layer - low temperature region and mix with cold fluid and thus transfer heat by **mixing motion**.

If the mixing motion of fluid particles takes place due to density difference caused by temperature difference, then this convection heat transfer is called **free convection (or) natural convection**.

If the motion of fluid particles is due to fan (or) pump (or) blower (or) any external means, then this convection heat transfer is called **forced convection**.

3. Radiation

Conduction and convection needs a medium for heat transfer, but radiation heat transfer takes place even in vacuum.

Radiation heat transfer occurs when the hot body and cold body are separated in space. The space may be filled up by a medium (or) vacuum.

Energy, emitted in the form of electromagnetic waves, by all bodies due to their temperatures is called thermal radiation.

GENERAL DIFFERENTIAL EQUATION OF HEAT CONDUCTION - CARTESIAN COORDINATES

Consider an infinitesimal rectangular element of sides dx, dy and dz as shown in **Fig. 1.9**.

Q_x = Rate of heat flow in x direction through the face $ABCD$

Q_{x+dx} = Rate of heat flow in x direction through the face $EFGH$

q_x = Heat flux $\left(\frac{Q_x}{A}\right)$ in x direction through face $ABCD$

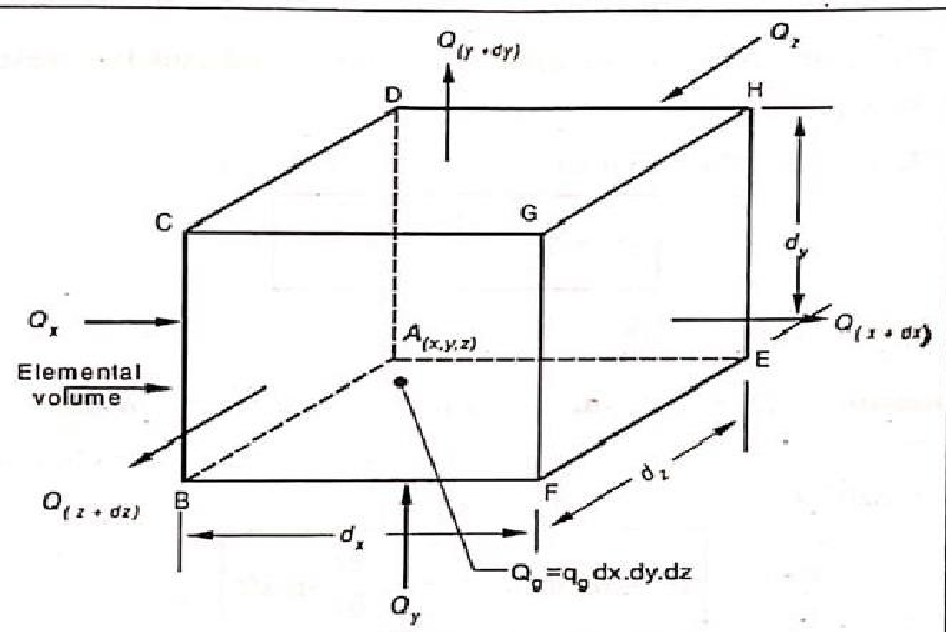


Fig. 1.9 Elemental volume for three-dimensional heat conduction analysis - Cartesian coordinates

q_{x+dx} = Heat flux $\left(\frac{Q_x + dx}{A}\right)$ in x direction through face $EFGH$

kx, ky, kz = Thermal conductivities along x, y and z axes

$\frac{\partial T}{\partial x}$ = Temperature gradient in x direction

The differential equation of conduction can be derived based on the law of conservation of energy (or) the first law of Thermodynamics. Let us apply the first law of thermodynamics to the control volume of **Fig. 1.9**.

$$\begin{bmatrix} \text{Quantity of} \\ \text{heat conducted} \\ \text{to the} \\ \text{elementary} \\ \text{volume in} \\ \text{face } ABCD \\ Q_x \end{bmatrix} + \begin{bmatrix} \text{Heat} \\ \text{generated} \\ \text{from inner} \\ \text{heat source} \\ \text{with in the} \\ \text{element} \\ Q_g \end{bmatrix} = \begin{bmatrix} \text{Change in} \\ \text{enthalpy} \\ \text{of element} \\ \frac{dh}{dt} \end{bmatrix} + \begin{bmatrix} \text{Work done} \\ \text{by} \\ \text{element} \\ W \end{bmatrix} \qquad ...(1.1)$$

The work done by an element is small and can be neglected in the above equation.

Hence, the above equation can be written as

$$\boxed{Q_x + Q_g = \frac{dh}{dt} + Q_{x+dx}} \qquad ...(1.2)$$

Now let us see one by one.

Qx: Quantity of heat conducted to the elementary volume

The rate of heat flow in to the element in x direction through the face $ABCD$ is

$$\boxed{Q_x = q_x \, dy \, dz = -k_x \frac{\partial T}{\partial x} dy \, dz} \qquad ...(1.3)$$

The rate of heat flow out of the element in x direction through he face $EFGH$ is

$$Q_{x+dx} = Q_x + \frac{\partial}{\partial x}(Q_x)\, dx$$

$$= -k_x \frac{\partial T}{\partial x} dy \, dz + \frac{\partial}{\partial x}\left[-k_x \frac{\partial T}{\partial x} dy \, dz \right] dx$$

$$\boxed{Q_{x+dx} = -k_x \frac{\partial T}{\partial x} dy \, dz - \frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x} \right] dx \, dy \, dz} \qquad ...(1.4)$$

$Q_x - Q_{x+dx}$ gives

$$Q_x - Q_{(x+dx)} = -k_x \frac{\partial T}{\partial x} dy\, dz - \left[-k_x \frac{\partial T}{\partial x} dy\, dz - \frac{\partial}{\partial x} \left[k_x \frac{\partial T}{\partial x} \right] dx\, dy\, dz \right]$$

$$= -k_x \frac{\partial T}{\partial x} dy\, dz + k_x \frac{\partial T}{\partial x} dy\, dz + \frac{\partial}{\partial x} \left[k_x \frac{\partial T}{\partial x} \right] dx\, dy\, dz$$

$$\Rightarrow Q_x - Q_{(x+dx)} = \frac{\partial}{\partial x} \left[k_x \frac{\partial T}{\partial x} \right] dx\, dy\, dz \quad \text{...(1.5)}$$

Similarly $Q_y - Q_{(y+dy)} = \frac{\partial}{\partial y} \left[k_y \frac{\partial T}{\partial y} \right] dx\, dy\, dz$ ...(1.6)

$$Q_z - Q_{(z+dz)} = \frac{\partial}{\partial z} \left[k_z \frac{\partial T}{\partial z} \right] dx\, dy\, dz \quad \text{...(1.7)}$$

Add (1.5) + (1.6) + (1.7)

$$\left.\begin{matrix}\text{Total heat conducted} \\ \text{in all direction}\end{matrix}\right\} = \frac{\partial}{\partial x} \left[k_x \frac{\partial T}{\partial x} \right] dx\, dy\, dz + \frac{\partial}{\partial y} \left[k_y \frac{\partial T}{\partial y} \right] dx\, dy\, dz + \frac{\partial}{\partial z} \left[k_z \frac{\partial T}{\partial z} \right] dx\, dy\, dz$$

Total heat conducted into the element from all directions

$$= \left[\frac{\partial}{\partial x} \left[k_x \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial y} \left[k_y \frac{\partial T}{\partial y} \right] + \frac{\partial}{\partial z} \left[k_z \frac{\partial T}{\partial z} \right] \right] dx\, dy\, dz \quad \text{...(1.8)}$$

Change in enthalpy of the element $\left(\frac{dh}{dt} \right)$

We know that,

$$\left\{\begin{matrix}\text{Change in} \\ \text{enthalpy} \\ \text{of the} \\ \text{element}\end{matrix}\right\} = \left\{\begin{matrix}\text{Mass} \\ \text{of the} \\ \text{element}\end{matrix}\right\} \times \left\{\begin{matrix}\text{Specific} \\ \text{heat} \\ \text{of the} \\ \text{element}\end{matrix}\right\} \times \left\{\begin{matrix}\text{Rise in} \\ \text{temperature} \\ \text{of element}\end{matrix}\right\}$$

$$= m \times C_p \times \frac{\partial T}{\partial t}$$

$$= (\rho \times dx\, dy\, dz) \times C_p \times \frac{\partial T}{\partial t}$$

[∵ Mass = Density × Volume

$$\left\{ \begin{array}{l} \textbf{Change in enthalpy of} \\ \textbf{the element} \end{array} \right\} = \rho\, C_p \frac{\partial T}{\partial t}\, dx\, dy\, dz \quad ...(1.9)$$

Heat generated from inner heat source within the element Q_g

Heat generated within the element is given by

$$Q_g = q_g\, dx\, dy\, dz \quad ...(1.10)$$

Substituting equation (1.8), (1.9) and (1.10) in equation (1.2)

$$\left[\frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial y}\left[k_y \frac{\partial T}{\partial y} \right] + \frac{\partial}{\partial z}\left[k_z \frac{\partial T}{\partial z} \right] \right] dx\, dy\, dz$$

$$+ q_g\, dx\, dy\, dz = \rho\, C_p \frac{\partial T}{\partial t}\, dx\, dy\, dz$$

$$\frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial y}\left[k_y \frac{\partial T}{\partial y} \right] + \frac{\partial}{\partial z}\left[k_z \frac{\partial T}{\partial z} \right] + q_g = \rho\, C_p \frac{\partial T}{\partial t}$$

When the material is isotropic,

$k_x = k_y = k_z = k =$ constant

$$\Rightarrow k\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] + q_g = \rho\, C_p \frac{\partial T}{\partial t}$$

Divided by k,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{k} = \frac{\rho\, C_p}{k} \frac{\partial T}{\partial t}$$

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}} \quad ...(1.11)$$

$$\nabla^2 T + \frac{q_g}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad ...(1.11)$$

It is a general three dimensional heat conduction equation in cartesian coordinates where, α = Thermal diffusivity $= \frac{k}{\rho C_p}$

Qx

Case (i)

When no internal heat generation is present

ie when $q_g = 0$, then the equation 1.11 becomes

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\nabla^2 T = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad \text{(Fourier equation)} \qquad ...(1.12)$$

Case (ii)

In steady state conditions, the temperature does not change with respect to time. Then the conduction takes place in the steady state ie $\frac{\partial T}{\partial t} = 0$. Hence the equation 1.11 becomes

$$\nabla^2 T + \frac{q_g}{k} = 0 \quad \text{(Poisson's equation)} \qquad ...(1.13)$$

Case (iii)

No heat generation; steady state conditions. Then the equation 1.11 becomes,

$$\nabla^2 T = 0 \quad \text{(Laplace equation)} \qquad ...(1.14)$$

Case (iv)

∴ Steady state, one-dimensional heat transfer,

$$\frac{\partial^2 T}{\partial x^2} + \frac{q_g}{k} = 0 \quad \text{...(1.15)}$$

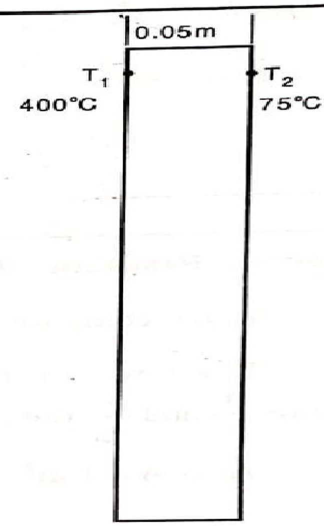
Case (v)

Steady state, one dimensional, without internal heat generation

$$\frac{\partial^2 T}{\partial x^2} = 0 \quad \text{...(1.16)}$$

Problem 1.1: *Find the rate of heat transfer per unit area through a copper plate 50 mm thick, whose one face is maintained at 400°C and otherface at 75°C. Take thermal conductivity of copper as* 370 W/m°C.

Solution



Given: $L = 0.05$ m ; $A = 1$ m^2

$k = 370$ W/m°C

$T_1 = 400$ °C; $T_2 = 75$ °C

$$\frac{Q}{A} = q = \frac{\Delta T}{R}$$

Refer **Pg. 44** of HMT Data book for formula.

$$R = \frac{L}{kA}$$

$$= \frac{0.05}{370 \times 1}$$

$$R = 1.351 \times 10^{-4} \text{ K/W}$$

$$q = \frac{\Delta T}{R} = \frac{T_1 - T_2}{R} = \frac{400 - 75}{1.351 \times 10^{-4}}$$

$$q = 2405.63 \text{ kW/m}^2$$

A stainless steel plate 2 cm thick is maintained at a temperature of 550°C at one face and 50°C on the other. The thermal conductivity of stainless steel at 300°C is 19.1 W/mK. Compute the heat transferred through the material per unit area.

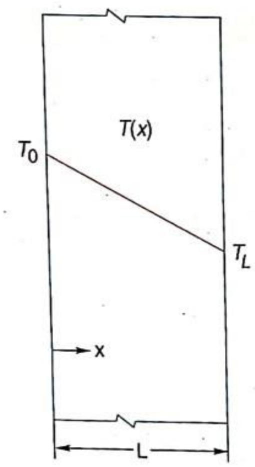


Fig. Ex. 1.1

Solution

This is the case of a plane wall as shown in Fig. Ex. 1.1. Using Eqn. (1.12).

$$Q_x = \frac{kA}{L}(T_0 - T_L)$$

or

$$\frac{Q_x}{A} = q_x = \frac{k}{L}(T_0 - T_L) = \frac{(19.1)(550 - 50)}{2 \times 10^{-2}} = 477.5 \text{ kW/m}^2$$

1.6 CONVECTION

For a fluid flowing at a mean temperature T_∞ over a surface at a temperature T_s (Fig. 1.5), Newton proposed the following heat convection equation:

$$q = Q/A = h(T_s - T_\infty) = h\Delta T \tag{1.14}$$

where q is the heat flux at the wall. The Eqn. (1.14) is called Newton's law of cooling. The heat transfer coefficient h has units $\text{W/m}^2\text{°C}$ or $\text{W/m}^2\text{ K}$ when the heat flux q is given in the units of W/m^2 and the temperature in °C.

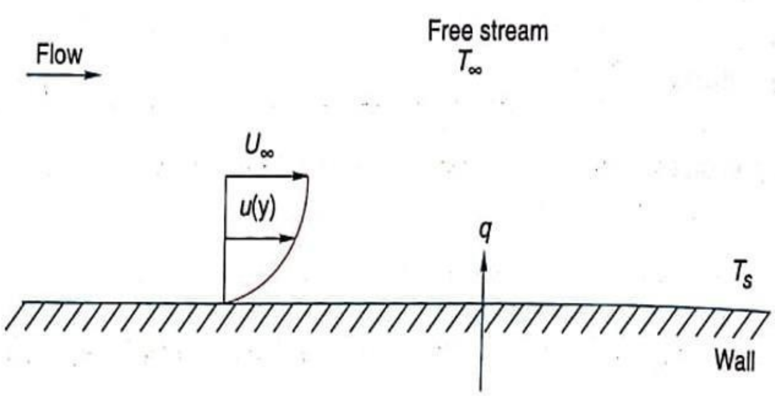


Fig. 1.5 Convection from a Heated Plate

Example 1.2

A flat plate of length 1 m and width 0.5 m is placed in an air stream at 30°C blowing parallel to it. The convective heat transfer coefficient is 30 W/m^2K. Calculate the heat transfer if the plate is maintained at a temperature of 300°C.

Solution

$$
\begin{aligned}
Q &= hA(T_s - T_\infty) \\
&= (30)\,(1.0)\,(0.5)\,(300 - 30) \\
&= 4.05 \text{ kW}.
\end{aligned}
$$

1.7 THERMAL RADIATION

According to the Stefan-Boltzmann law, the radiation energy emitted by a body is proportional to the *fourth power* of its absolute temperature.

$$Q = \sigma A T_1^4 \tag{1.16}$$

where σ is called the *Stefan-Boltzmann constant* with the value of 5.6697×10^{-8} W/m^2K^4, and T_1 is the surface temperature in Kelvin.

Consider a black body (a perfect emitter and perfect absorber) of surface area A_1 and at an absolute temperature T_1 exchanging radiation with another black body (similar) at a temperature T_2. The net heat exchange is proportional to the difference in T^4.

$$Q = \sigma A_1 (T_1^4 - T_2^4) \tag{1.17}$$

The real surfaces, like a polished metal plate, do not radiate as much energy as a black body. The 'gray' nature of real surfaces can be accounted for by introducing a factor ε_1 in Eqn. (1.17) called *emissivity* which relates radiation between gray and black bodies at the same temperature.

$$Q = \sigma A_1 \varepsilon_1 (T_1^4 - T_2^4) \tag{1.18}$$

To account for geometry and orientation of two black surfaces exchanging radiation Eqn. (1.17) is modified to

$$Q = \sigma A_1 \varepsilon_1 F (T_1^4 - T_2^4) \tag{1.19}$$

where the factor *F*, called *view factor*, is dependent upon geometry of the two surfaces exchanging radiation, see Planck (1959).

Example 1.3

A 'radiator' in a domestic heating system operates at a surface temperature of 55°C. Determine the rate

Example 1.3

A 'radiator' in a domestic heating system operates at a surface temperature of 55°C. Determine the rate at which it emits radiant heat per unit area if it behaves as a black body.

Solution

$$\frac{Q}{A} = q = \sigma T^4 = 5.6697 \times 10^{-8} \times (273 + 55)^4 = 0.66 \text{ kW/m}^2$$

1.9 COMBINED MECHANISMS OF HEAT TRANSFER

It is not unusual to observe that the heat transfer is taking place due to two, or perhaps all three, mechanisms. The most frequently encountered instance is one in which a solid wall (usually plane or cylindrical) separates two convicting fluids, *e.g.*, the tubes of a heat exchanger. As mentioned earlier, the steam generating tubes of a boiler receive heat from the products of combustion by all three modes of heat transfer.

The overall heat transfer by combined modes is usually expressed in terms of an overall conductance or 'overall heat transfer coefficient' U, defined by the relation:

$$Q = UA\,(\Delta T) \tag{1.27}$$

The overall heat transfer coefficient is a quantity such that the rate of heat flow through a configuration is given by taking a product of U, the surface area and the overall temperature difference.

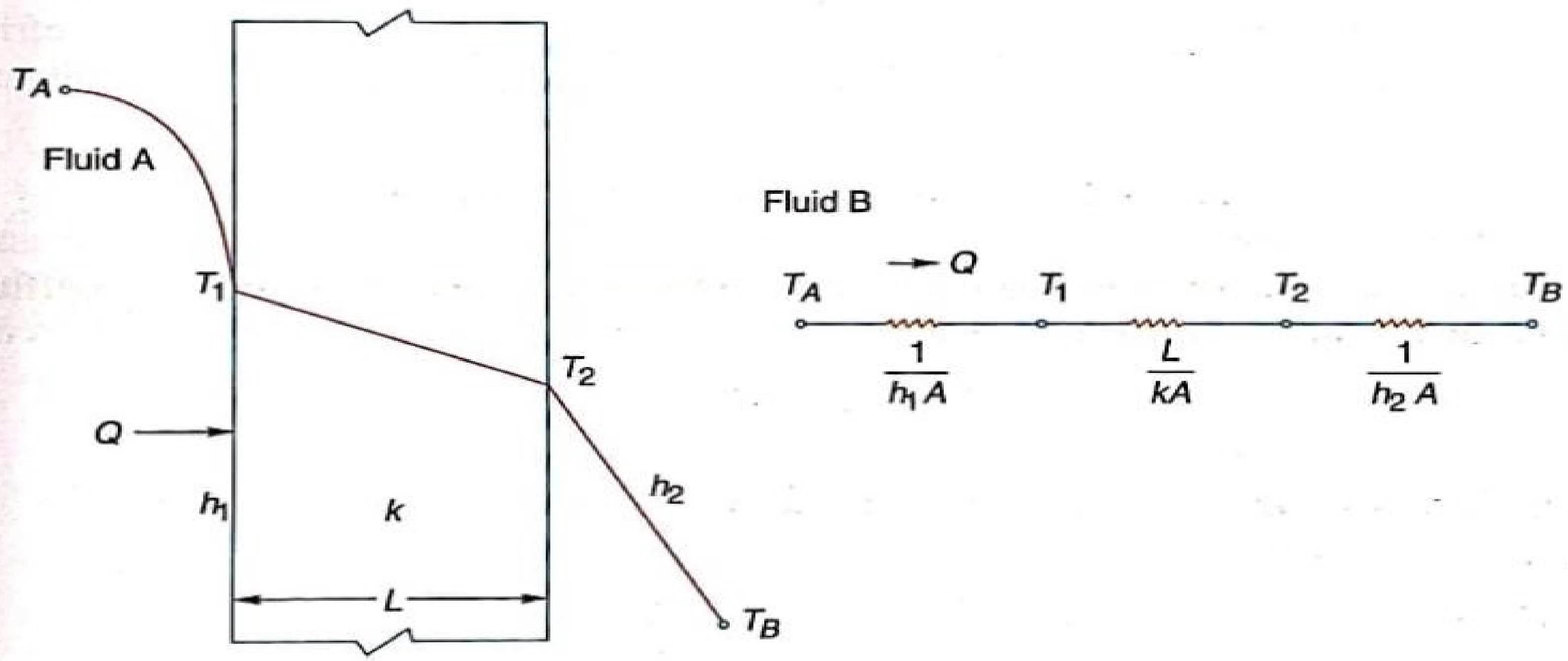


Fig. 1.7 Overall Heat Transfer through a Plane Wall with Resistance Analogy

In the case of a plane wall shown in Fig. 1.7 heated on one side by a hot fluid A and cooled on the other side by a cold fluid B, the heat transfer rate is given by:

$$Q = h_1 A(T_A - T_1) = \frac{kA}{L}(T_1 - T_2) = h_2 A(T_2 - T_B)$$

from which,

$$T_A - T_1 = \frac{Q}{h_1 A}$$

$$T_1 - T_2 = \frac{Q}{\frac{kA}{L}}$$

$$T_2 - T_B = \frac{Q}{h_2 A}$$

Adding these equations we eliminate the unknown temperatures T_1 and T_2 to give the solution for heat flow as

$$Q = \frac{T_A - T_B}{(1/h_1 A) + (L/kA) + (1/h_2 A)} \quad (1.28)$$

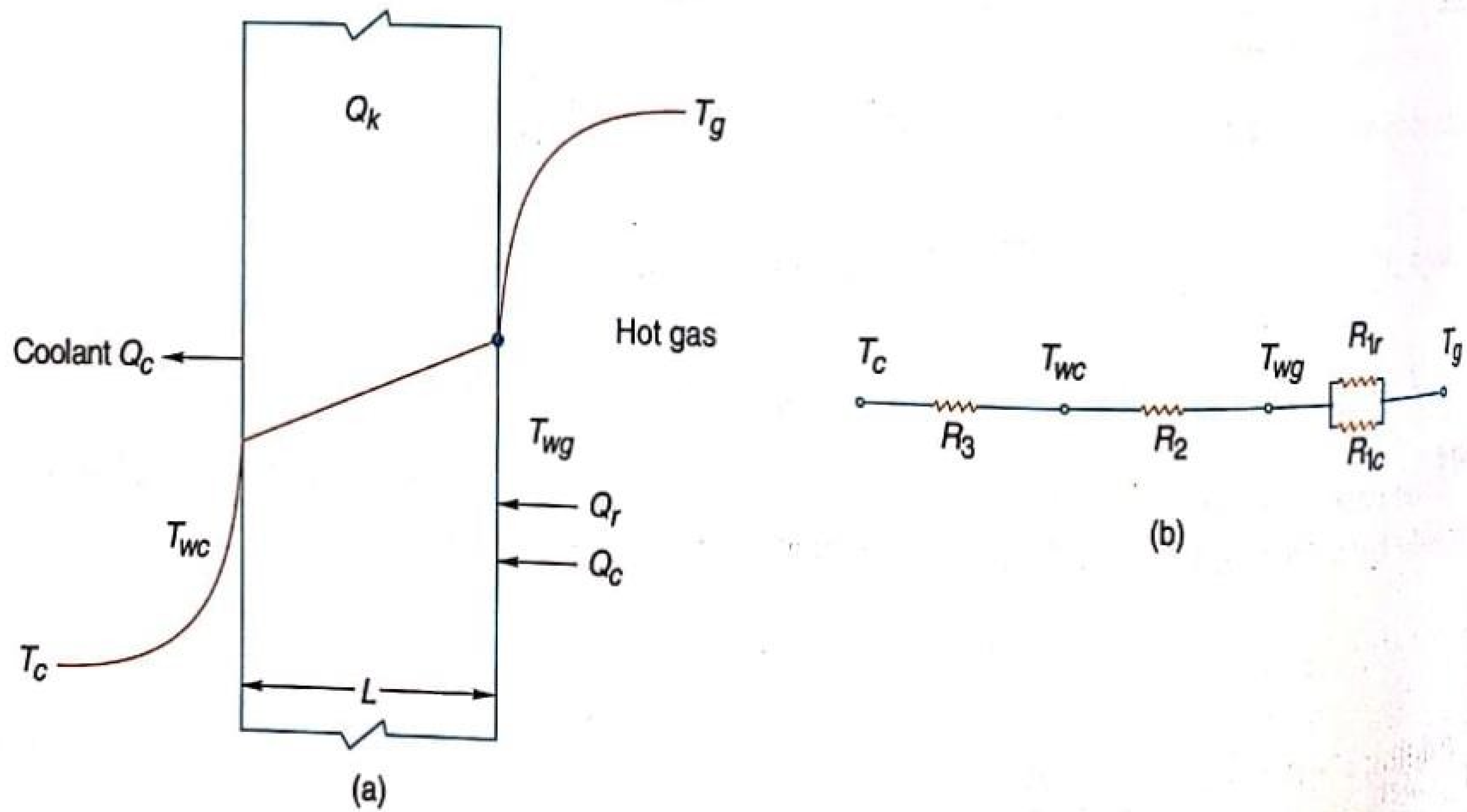
remembering that the value $(1/hA)$ is used to represent the convection resistance and (L/kA) is the conduction resistance. In accordance with Eqn. (1.27), the overall heat transfer coefficient is:

$$U = \frac{1}{1/h_1 + L/k + 1/h_2} = \frac{1}{A \sum R_{th}} \quad (1.29)$$

The overall coefficient depends upon the geometry of the separating wall, its thermal properties, and the convective coefficients at the two surfaces. The overall heat transfer coefficient is particularly useful in the case of composite walls, such as in the design of structural walls for boilers, refrigerators, air-conditioned buildings, etc. Use of overall heat transfer coefficient is also made of in the design of heat exchangers.

Example 1.4

The inner surface of a combustion chamber wall receives heat from the products of combustion. The wall is being cooled by a coolant on the outer side. Compute the overall heat transfer coefficient and draw the equivalent thermal circuit.

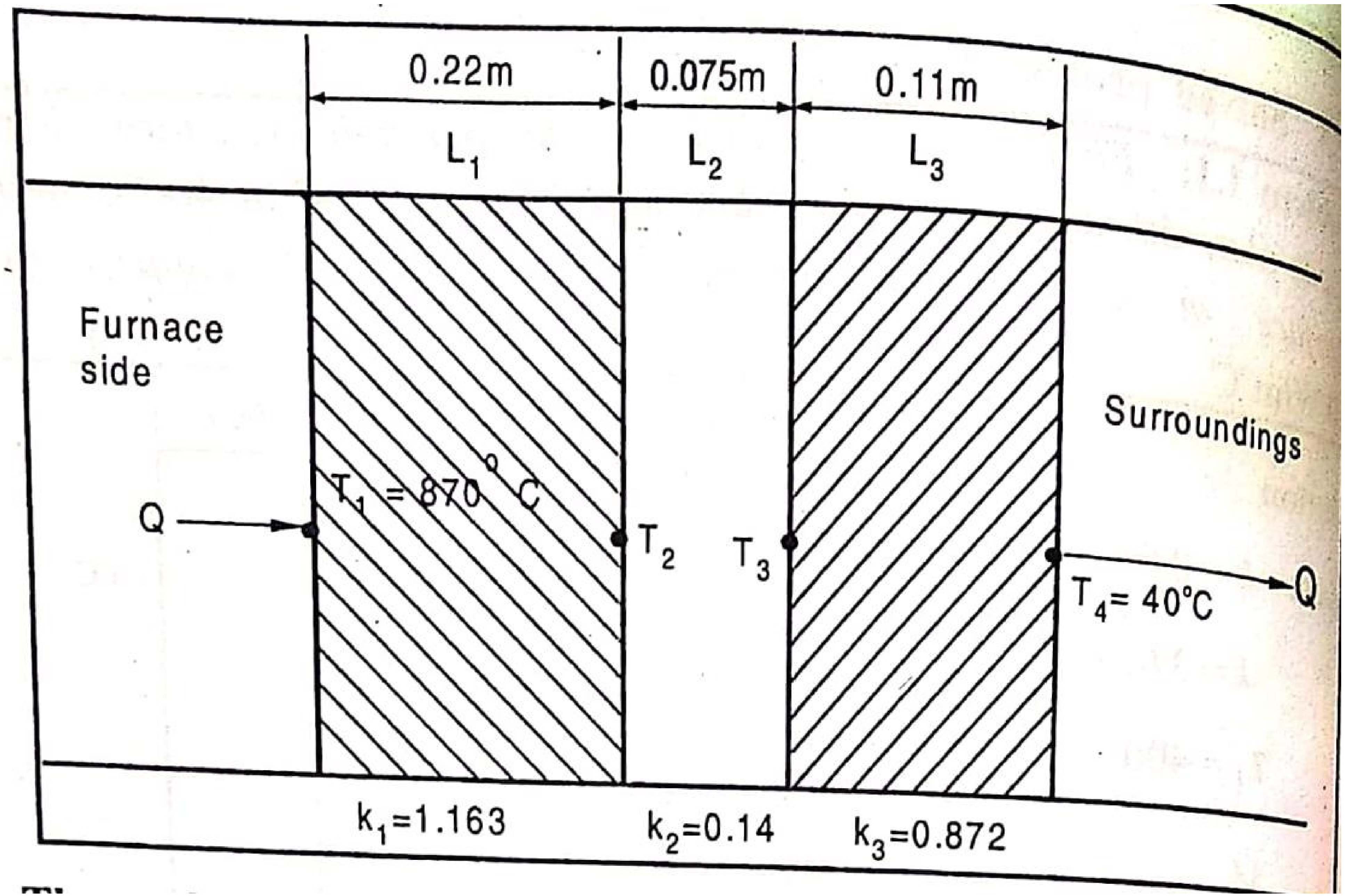


Problem 1.2: *A furnace wall is made up of three layers, one is fire brick, one is insulating layer and one is red brick. The inner and outer surfaces temperature are at* 870°C and 40°C *respectively. The respective conductive heat transfer coefficients of the layers are 1.163, 0.14 and 0.872* W/m°C *and the thicknesses are 22 cm, 7.5 cm and 11 cm. Find the rate of heat loss per sq. meter and the interface temperatures.*

Solution

From **Pg. 45**-CPK-Data book

$$Q = \frac{\Delta T}{R}$$


0.22m
0.075m
0.11m
L1
L2
L3
Furnace side
Surroundings
Q
T1 = 870°C
T2
T3
T4= 40°C
Q
k1=1.163
k2=0.14
k3=0.872

Thermal Resistance *R*

$$R \text{ for composite wall } = \frac{1}{A}\left[\frac{1}{h_a} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_b}\right]$$

Since we are not considering convective heat transfer, we can ignore $\frac{1}{h_a}$ and $\frac{1}{h_b}$ (i.e., $\frac{1}{h_a} = 0$ and $\frac{1}{h_b} = 0$)

Also $A = 1 \text{ m}^2$

$$\text{So } R = \frac{1}{A}\left[\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}\right]$$

$$= \frac{1}{1}\left[\frac{0.22}{1.163} + \frac{0.075}{0.14} + \frac{0.11}{0.872}\right]$$

$$= 0.8510 \text{ K/W}$$

$$Q = \frac{T_1 - T_4}{R} = \frac{(870 - 40)}{0.8510} = \frac{830}{0.8510} = \mathbf{975.3 \text{ W}}$$

$$q = \frac{Q}{A} = \frac{Q}{1} = \mathbf{975.3 \text{ W/m}^2}$$

$$R_1 = \frac{L_1}{k_1 A_1} = \frac{0.22}{1.163 \times 1}$$

$$= 0.1892$$

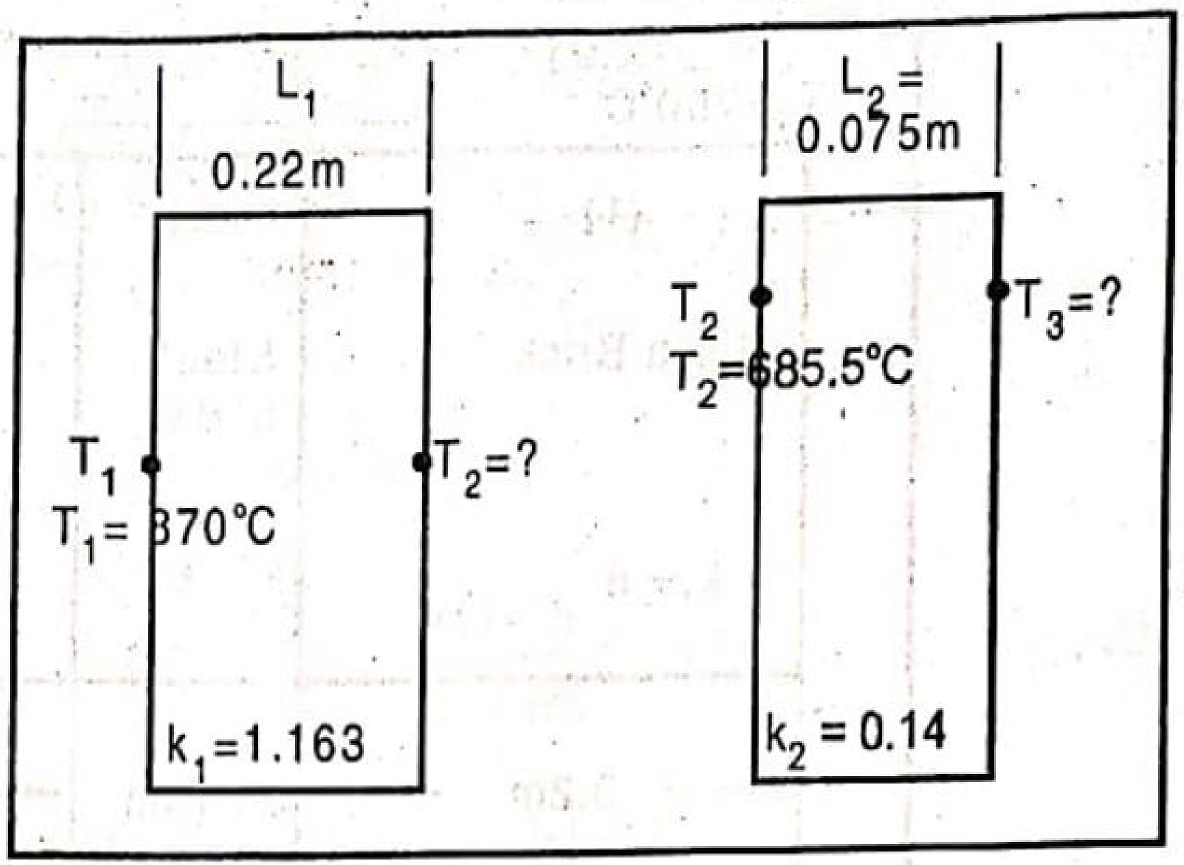
$$870 - T_2 =$$

$$975.3 \times 0.1892 = 184.5$$

$$T_2 = 870 - 184.5$$

$$= 685.5° \text{ C}$$

$$\mathbf{T_2 = 685.5\ °\ C}$$



Similarly,

$$\Delta T_2 = T_2 - T_3$$

$$= Q \times R_2$$

$$R_2 = \frac{L_2}{k_2 A_2} = \frac{0.075}{0.14 \times 1} = 0.5357$$

$$T_2 - T_3 = Q \times R_2$$

$$685.51 - T_3 = 975.3 \times 0.5357$$

$$T_3 = 685.51 - (975.3 \times 0.5357)$$

$$\boxed{\mathbf{T_3 = 163.03^\circ\ C}}$$

1.5 GENERAL DIFFERENTIAL EQUATION OF HEAT CONDUCTION - CYLINDRICAL COORDINATES

The heat conduction equation in cartesian coordinates can be used for rectangular solids like slabs, cubes, etc. But for cylindrical shapes like rods and pipes, it is convenient to use cylindrical coordinates. Fig. **1.10** shows a cylindrical coordinate system for general conduction equation.

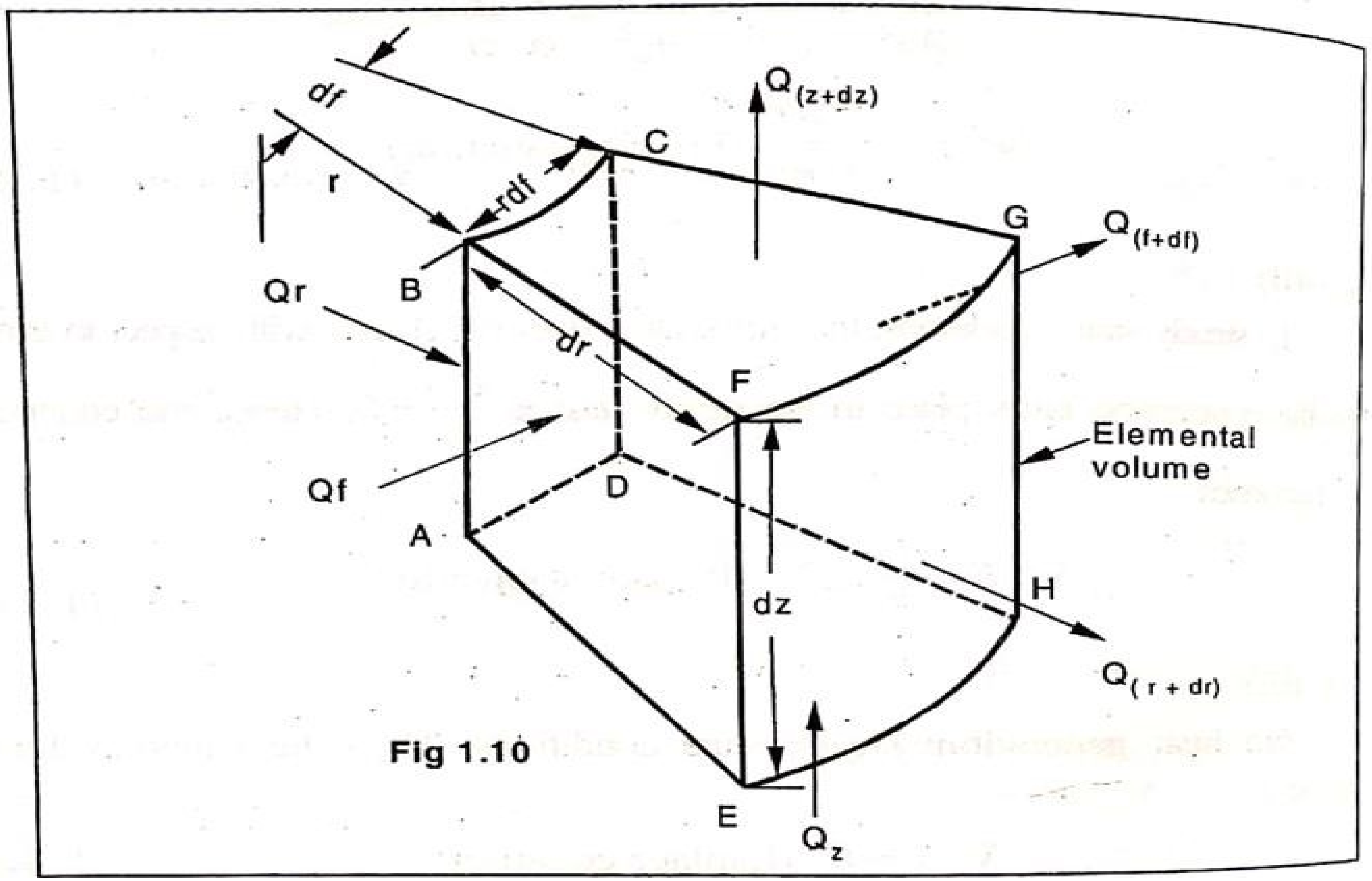


Fig 1.10

Q_r = Heat conducted to the element in the 'r'

direction through left face $ABCD$

Q_g = Heat generated with in the element

$\frac{dh}{dt}$ = Change in ethalpy per unit time

Q_{r+dr} = Heat conducted out of the element in 'r'

direction through the right face $EFGH$

1.7.2 Heat conduction through composite walls with fluid on both sid (with inside and outside convection)

A composite wall is composed of several different layers, eac having a different thermal conductivity. Consider a composite wall mac up of three parallel layers as shown in **Figure 1.14(a)**.

Since the rate of heat transfer through each layer (slab) is sam we have

$$Q = \frac{k_1 A (T_1 - T_2)}{L_1} = \frac{k_2 A (T_2 - T_3)}{L_2} = \frac{k_3 A (T_3 - T_4)}{L_3} \quad \text{...(1.37}$$

In most of the science and engineering applications, fluid flow on both sides of the composite walls.

Hence, we should consider convection on both sides. Then,

$$Q = h_a A (T_a - T_1) = \frac{k_1 A (T_1 - T_2)}{L_1} = \frac{k_2 A (T_2 - T_3)}{L_2}$$

$$= \frac{k_3 A (T_3 - T_4)}{L_3} = A h_b (T_4 - T_b)$$

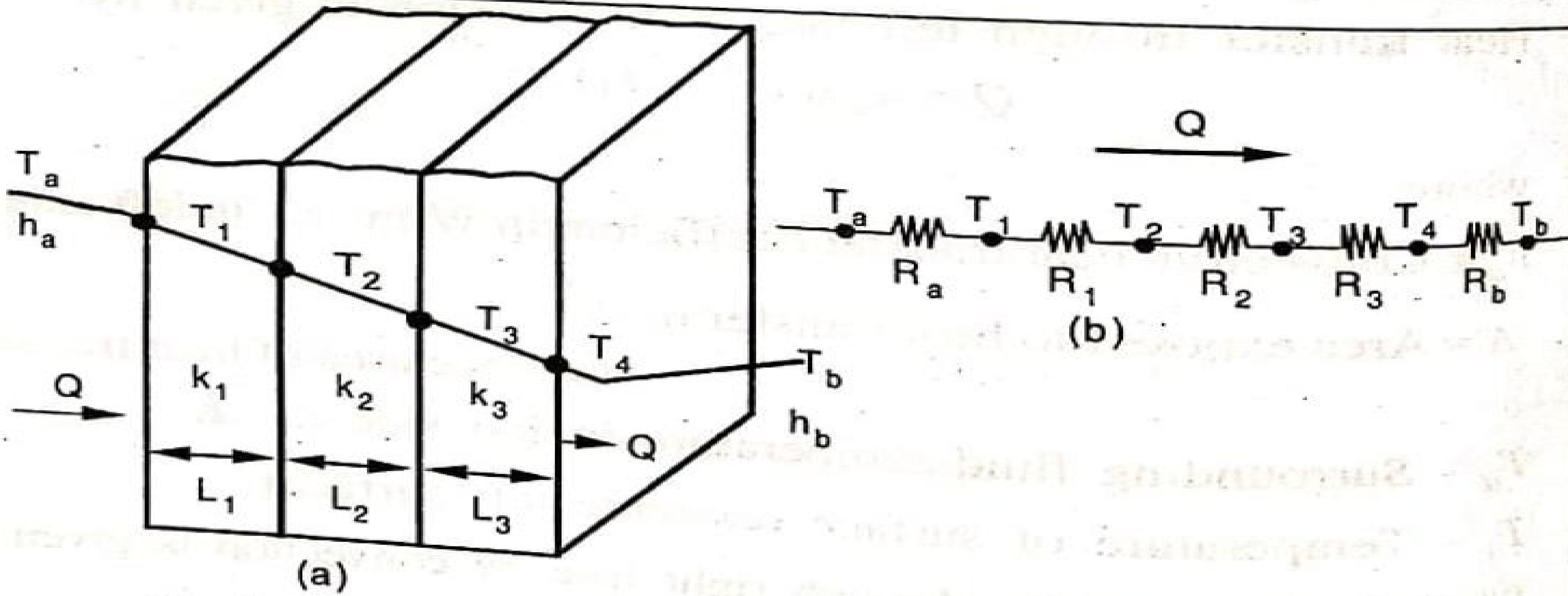


Fig. 1.14 (a) conduction through a composite slab with fluid on both sides and (b) equivalent thermal resistance circuit

Equation (1.38) can be written as ...(1.38)

$$(T_a - T_1) = \frac{Q}{h_a A} = QR_a \quad ...(1.39)$$

$$(T_1 - T_2) = \frac{QL_1}{k_1 A} = QR_1 \quad \text{...(1.40)}$$

$$(T_2 - T_3) = \frac{QL_2}{k_2 A} = QR_2 \quad \text{...(1.40)}$$

$$(T_3 - T_4) = \frac{QL_3}{k_3 A} = QR_3 \quad \text{...(1.41)}$$

$$(T_4 - T_b) = \frac{Q}{h_b A} = QR_b \quad \text{...(1.42)}$$

where, R_a and R_b are the thermal resistance of convection. Adding **Eqs.** from **1.38** to **1.42,** we get

$$T_a - T_b = Q\,(R_a + R_1 + R_2 + R_3 + R_b)$$

or

$$Q = \frac{T_a - T_b}{R_a + R_1 + R_2 + R_3 + R_b} \quad \text{...(1.43)}$$

For n number of slabs,

$$Q = \text{Heat flow} = \frac{T_a - T_b}{R_a + R_b + \sum_{i=1}^{n} R_i}$$

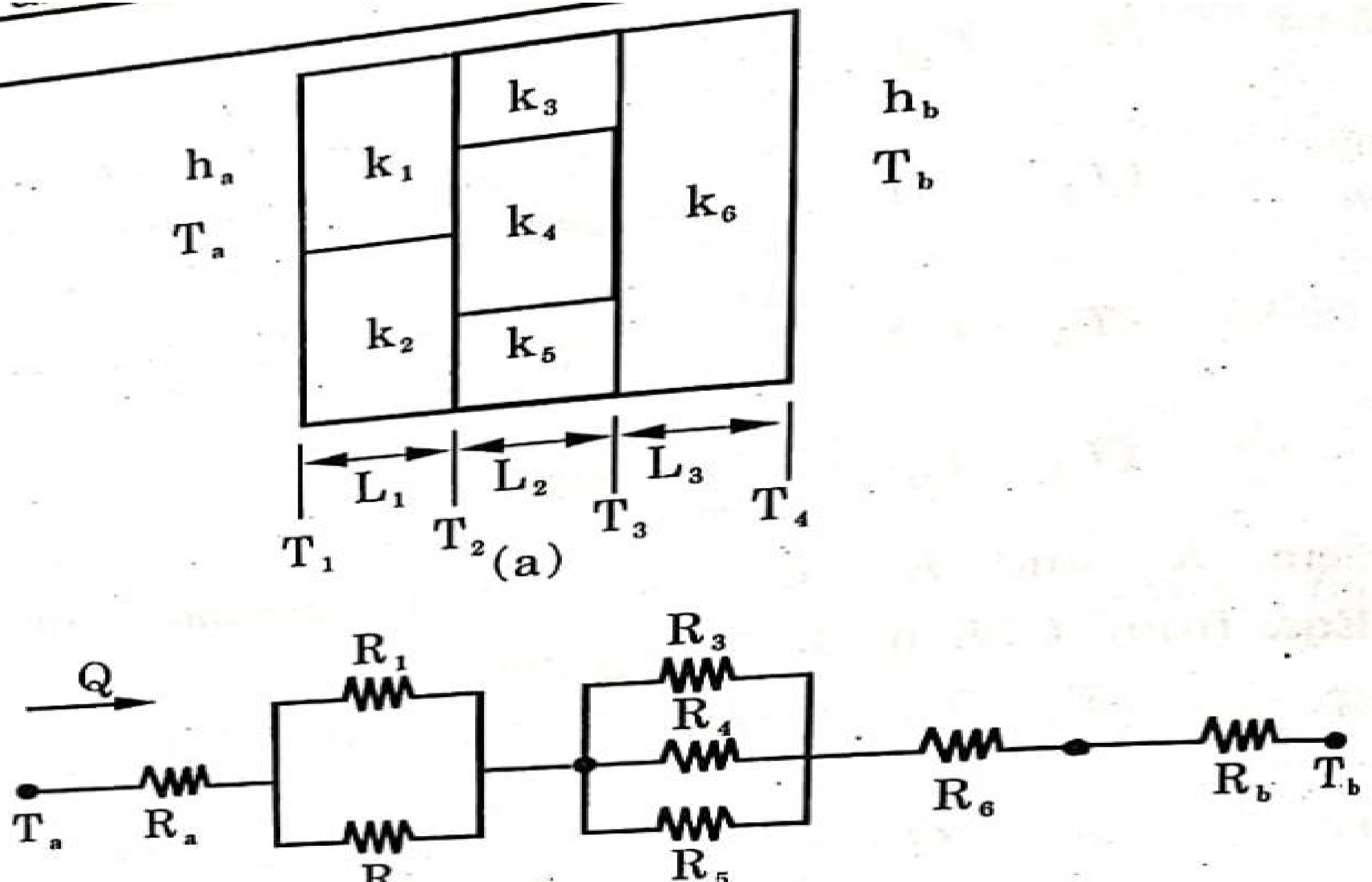
$$= \frac{\text{Overall temperature difference}}{\text{Thermal resistance}} = \frac{\Delta T_0}{\sum R} \quad \text{...(1.44)}$$

[Refer HMT Data book Page 45 for formula]

$$\therefore \; Q = \frac{A\,(T_a - T_b)}{\left(\frac{1}{h_a} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_b}\right)} = \frac{A\,(T_a - T_b)}{\left(\frac{1}{h_a} + \frac{1}{h_b} + \sum_{i=1}^{n} \frac{L_i}{k_i}\right)}$$

$$= UA\,(T_a - T_b) \quad \text{...(1.45)}$$

where U = overall heat transfer coefficient



(Refer page 47 of HMT data book)

(b)

Fig. 1.9. (a) A mix of series and parallel composite walls and (b) the equivalent thermal resistance circuit

If the heat transfer by convection on the two sides of the composite wall is absent, then

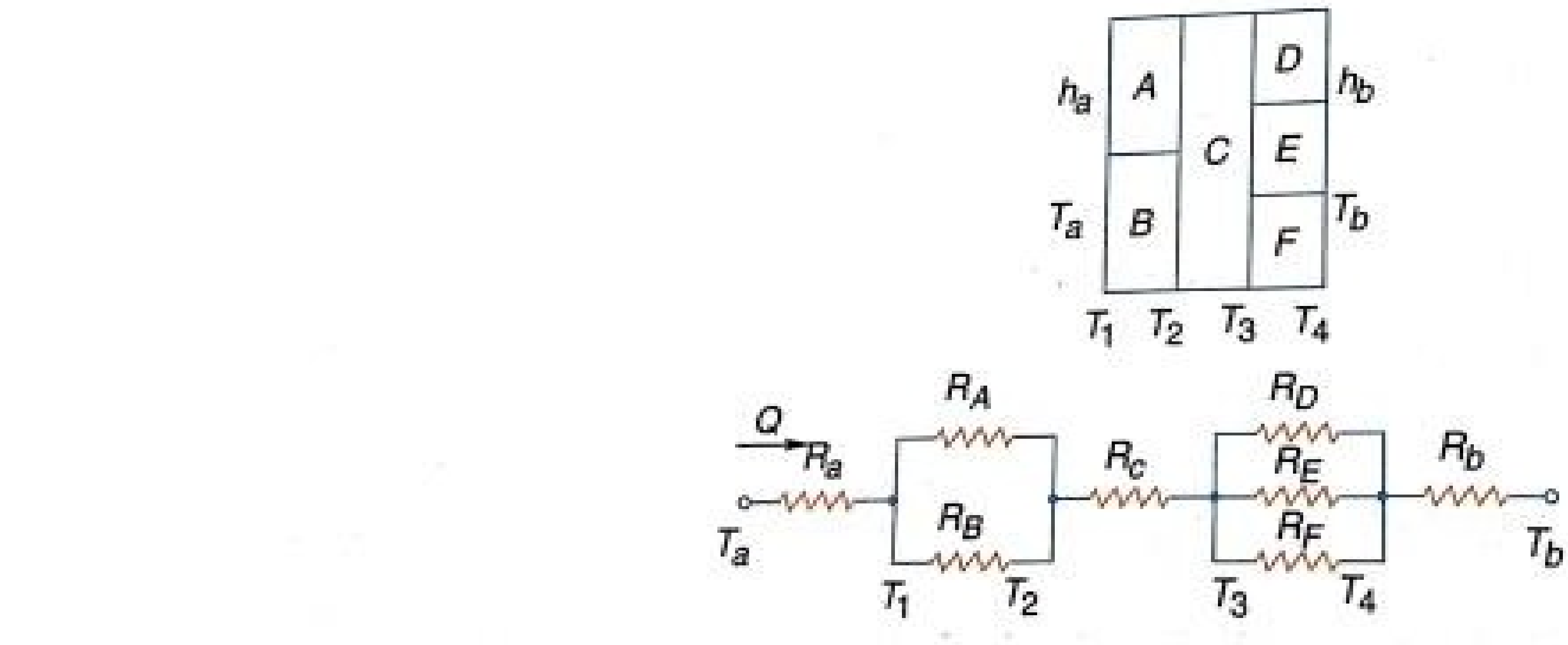


Fig. 3.6 Series and Parallel Composite Wall and its Thermal Circuit

$$Q = \frac{T_1 - T_{n+1}}{\frac{1}{A}\sum \frac{L_n}{k_n}}$$

The electrical analogy method may be used to solve complex problems involving both series and parallel thermal resistances. One such typical problem with its thermal circuit is shown in Fig. 3.6.

Here

$$\sum R = R_a + \frac{1}{\left(\frac{1}{R_A} + \frac{1}{R_B}\right)} + R_c + \frac{1}{\left(\frac{1}{R_D} + \frac{1}{R_E} + \frac{1}{R_F}\right)} + R_b = \frac{1}{UA}$$

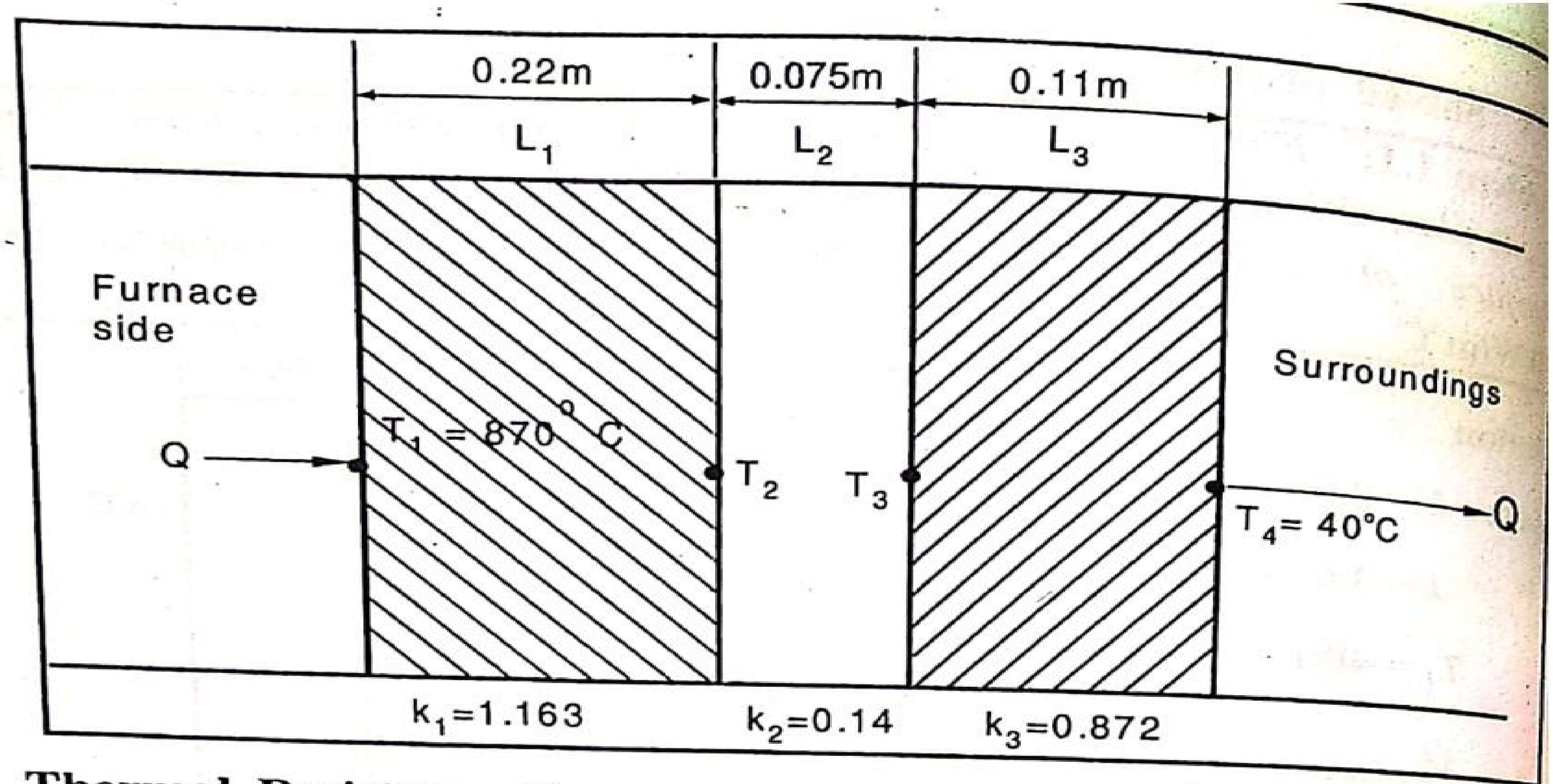
$\therefore$ $$Q = \frac{T_a - T_b}{\sum R}$$

Problem 1.2: *A furnace wall is made up of three layers, one is fire brick, one is insulating layer and one is red brick. The inner and outer surfaces temperature are at* 870°C and 40°C *respectively. The respective conductive heat transfer coefficients of the layers are 1.163, 0.14 and 0.872* W/m°C *and the thicknesses are 22 cm, 7.5 cm and 11 cm. Find the rate of heat loss per sq. meter and the interface temperatures.*

Solution

From **Pg. 45**-CPK-Data book

$$Q = \frac{\Delta T}{R}$$



Thermal Resistance *R*

$$R \text{ for composite wall } = \frac{1}{A}\left[\frac{1}{h_a} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_b}\right]$$

Since we are not considering convective heat transfer, we can ignore $\frac{1}{h_a}$ and $\frac{1}{h_b}$ (i.e., $\frac{1}{h_a} = 0$ and $\frac{1}{h_b} = 0$)

Also $A = 1 \text{ m}^2$

$$\text{So } R = \frac{1}{A}\left[\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}\right]$$

$$= \frac{1}{1}\left[\frac{0.22}{1.163} + \frac{0.075}{0.14} + \frac{0}{}\right.$$

$$= 0.8510 \text{ K/W}$$

$$Q = \frac{T_1 - T_4}{R} = \frac{(87}{}$$

$$q = \frac{Q}{A} =$$

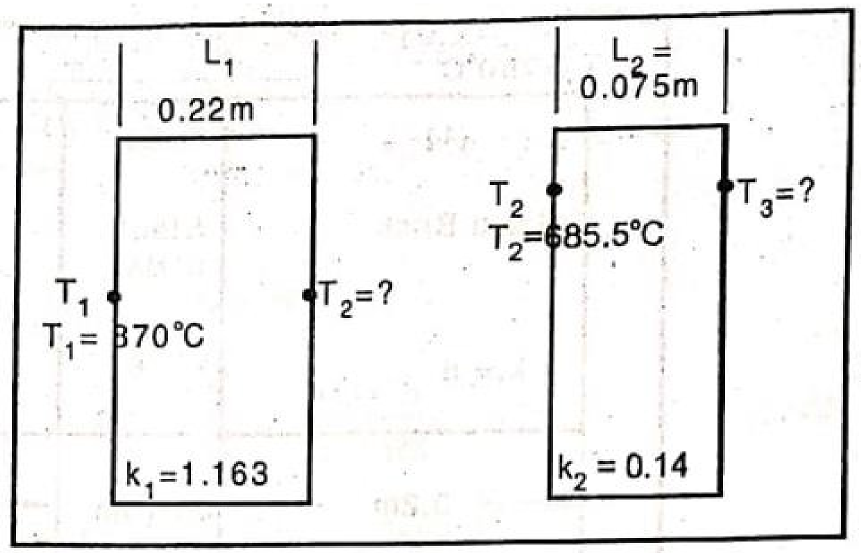
For Interfac

F

$(T$

$$R_1 = \frac{L_1}{k_1 A_1} = \frac{0.22}{1.163 \times 1}$$

$= 0.1892$



$870 - T_2 =$

$975.3 \times 0.1892 = 184.5$

$$T_2 = 870 - 184.5$$

$$= 685.5° \text{ C}$$

$$\mathbf{T_2 = 685.5\ ° \ C}$$

Similarly,

$$\Delta T_2 = T_2 - T_3$$

$$= Q \times R_2$$

$$R_2 = \frac{L_2}{k_2 A_2} = \frac{0.075}{0.14 \times 1} = 0.5357$$

$$T_2 - T_3 = Q \times R_2$$

$$685.51 - T_3 = 975.3 \times 0.5357$$

$$T_3 = 685.51 - (975.3 \times 0.5357)$$

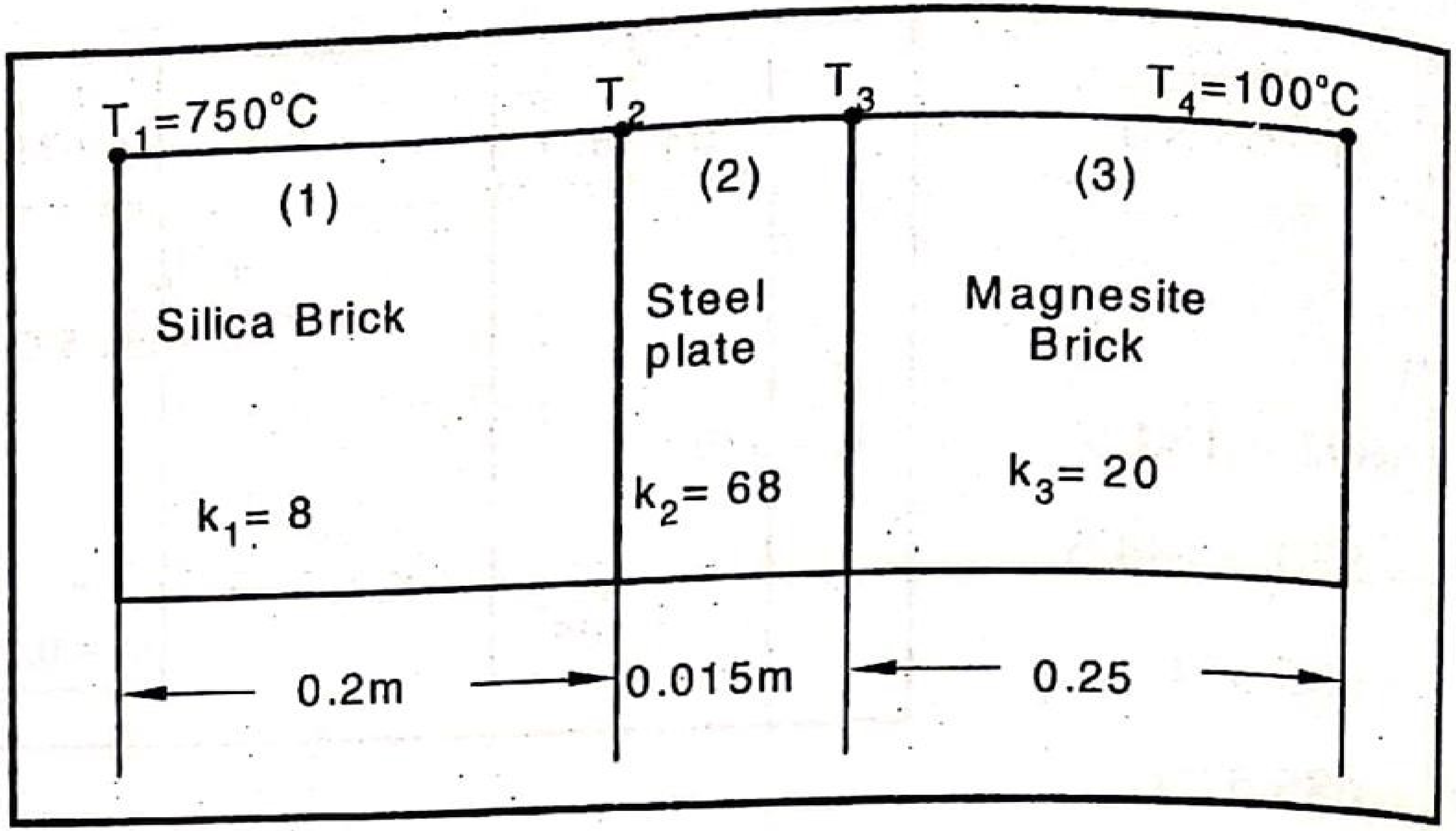
$$\boxed{\mathbf{T_3 = 163.03° \ C}}$$

Problem 1.3: *A composite wall is made of 15 mm thick of steel plate lined inside with Silica brick-200 mm thick and on the outside magnesite brick-250 mm thick. The inner and outer surface temperature are* 750°C and 100°C *respectively. The k for silica, steelplate and magnesite brick are* $8\ \frac{W}{m°C}$, $68\ \frac{W}{m°C}$ and $20\ \frac{W}{m°C}$ *respectively. Determine heat flux, interface temperatures.*

Solution

$$\text{Heat flux} = \frac{Q}{A} = q$$

From **Pg 45** of CPK



$$R = \frac{1}{A}\left[\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}\right] \qquad \left[\because \frac{1}{h_a} = 0 \text{ and } \frac{1}{h_b} = 0\right.$$

$$= \frac{1}{1}\left[\frac{0.2}{8} + \frac{0.015}{68} + \frac{0.25}{20}\right] = \mathbf{0.03772\ K/W}$$

$$Q = \frac{(\Delta T)_{overall}}{R}$$

$$Q = \frac{(\Delta T)_{overall}}{R}$$

$$= \frac{(T_1 - T_4)}{R} = \frac{750 - 100}{0.03772} = 17232 \text{ W}$$

$$\mathbf{Q = 17232 \ W}$$

To Find Interface Temperatures

To find T_2

$$\Delta T_1 = Q \times R_1$$

$$T_1 - T_2 = 17232 \times R_1$$

$$R_1 = \frac{1}{A}\left[\frac{L_1}{k_1}\right] \qquad [\because A = 1 \text{ m}^2]$$

$$= \frac{1}{1}\left[\frac{0.2}{8}\right] = 0.025 = \text{K/W}$$

$$T_1 - T_2 = 750 - T_2 = Q \times R_1$$

$$= 17232 \times 0.025$$

$$T_2 = 750 - (17232 \times 0.025) = 319.2°C$$

$$\boxed{T_2 = 319.2°C.}$$

To find T_3

$$T_2 - T_3 = Q \times R_2$$

$$R_2 = \frac{L_2}{Ak_2} = \frac{0.015}{1 \times 68} = 2.205 \times 10^{-4} \text{ K/W}$$

$$319.2 - T_3 = 17232 \times 2.205 \times 10^{-4}$$

$$T_3 = 319.2 - (17232 \times 2.205 \times 10^{-4}) = = 315.4 ° C$$

$$\boxed{T_3 = 315.4°C}$$

Alternate Method : To find T_3

$$Q = \frac{T_1 - T_3}{R_1 + R_2}$$

$$17232 = \frac{750 - T_3}{0.025 + 2.205 \times 10^{-4}}$$

$$17232\,(0.025 + 2.205 \times 10^{-4}) = 750 - T_3$$

$$T_3 = 750 - [\,17232(0.025 + 2.205 \times 10^{-4})\,]$$

$$= \mathbf{315.4°C.}$$

To find T_2

$$\left[R_3 = \frac{0.25}{1 \times 20} = 0.0125 \right]$$

$$Q = \frac{T_2 - T_4}{R_2 + R_3}$$

$$17232 = \frac{T_2 - 100}{2.205 \times 10^{-4} + 0.0125}$$

$$17232(2.205 \times 10^{-4} + 0.0125) = T_2 - 100$$

$$T_2 = 17232(2.205 \times 10^{-4} + 0.0125) + 100$$

$$= \mathbf{319.19^\circ C.}$$

Problem 1.4: *The temperature distribution through a furnace wall consisting of fire brick, block insulation and steel plate is given below. Determine heat flux, thermal conductivity of block insulation and steel plate, heat transfer coefficient for gas side and air side.*

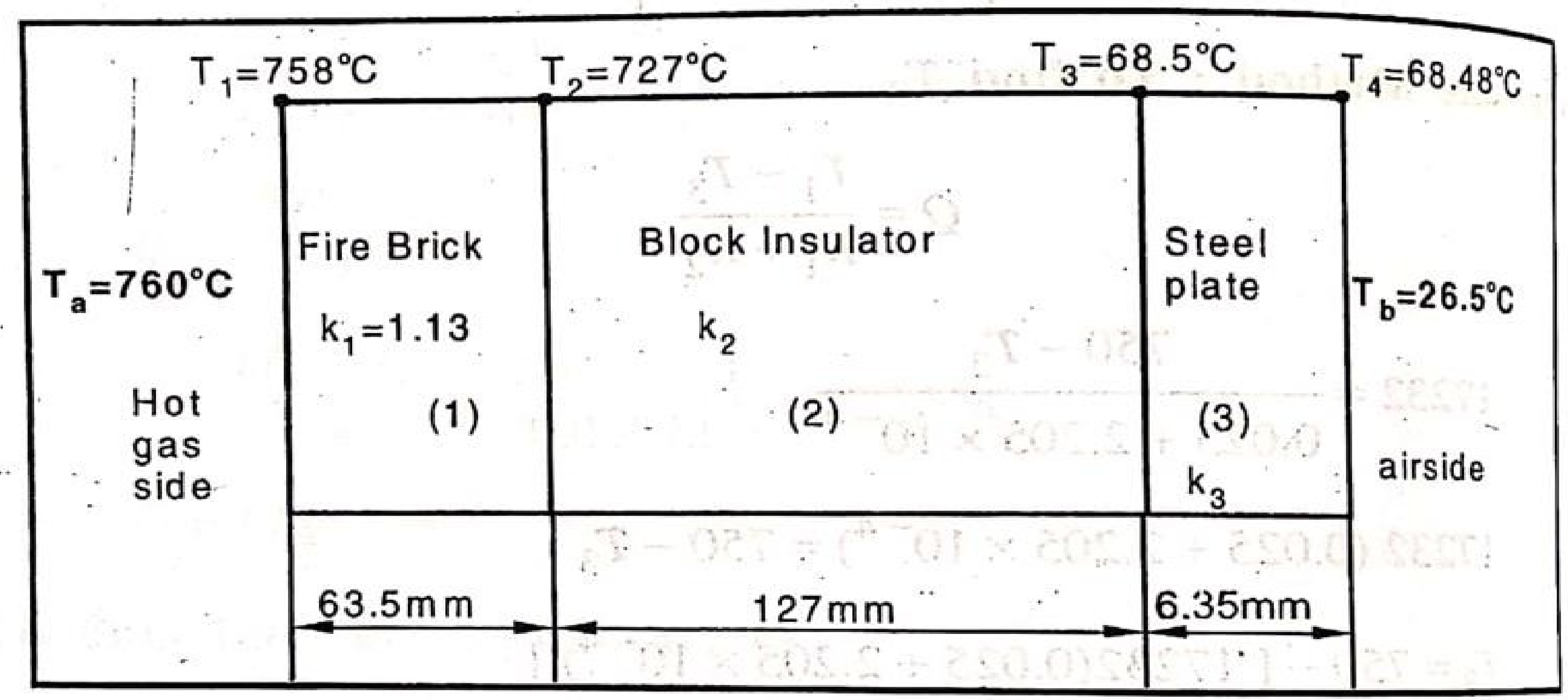
Solution

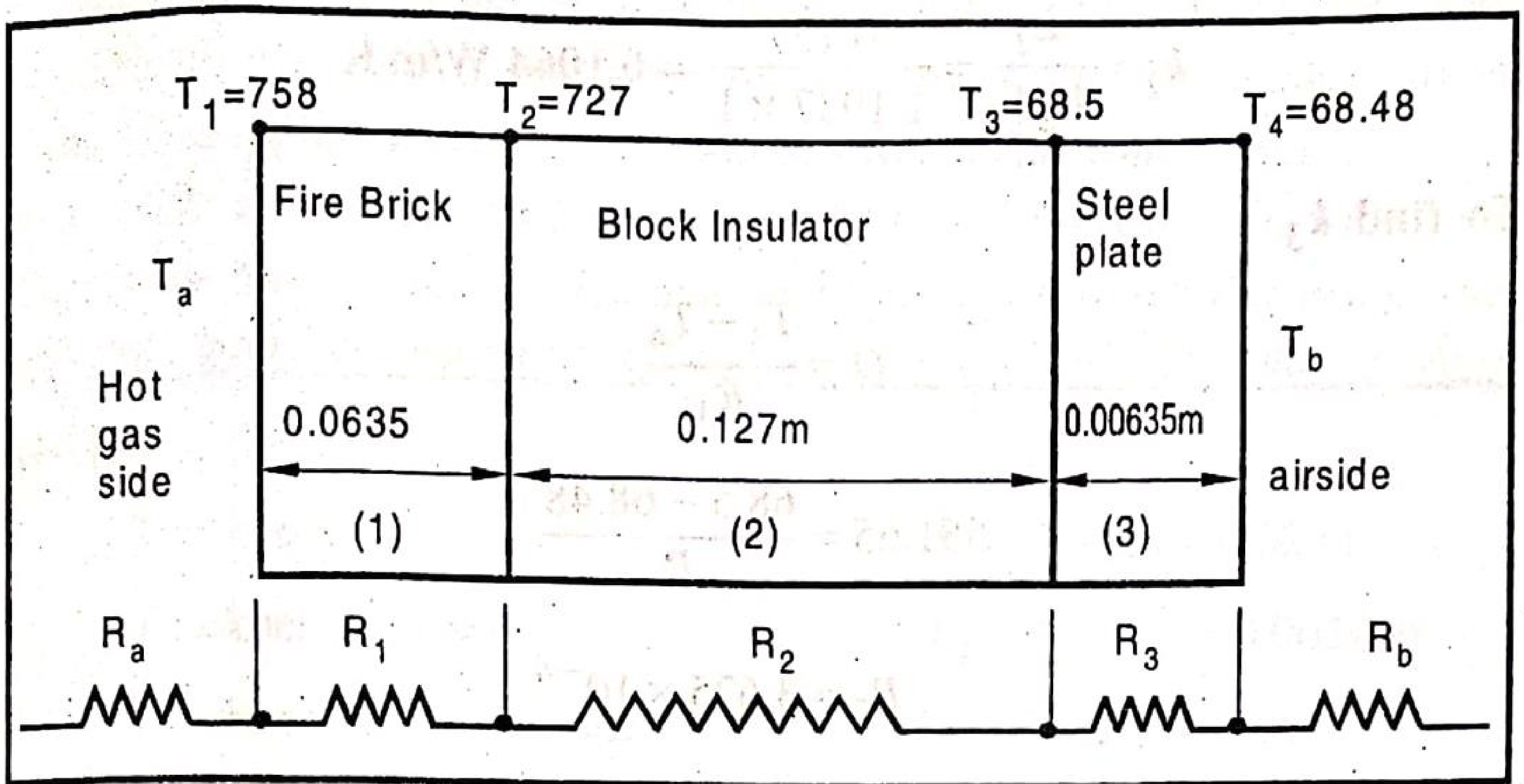
Note

From hot gas side to fire brick, heat is transferred by convection.

So

$$Q_{convection} = h_a A (T_a - T_1)$$





$$\text{where } R \text{ for composite wall} = \frac{1}{A}\left[\frac{1}{h_a} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_b}\right]$$

$$A = 1 \text{ m}^2 \text{ (not given)}$$

To find Q

$$Q = \frac{T_1 - T_2}{R_1}$$

$$R_1 = \frac{L_1}{k_1 A_1} = \frac{0.0635}{1.13 \times 1} = 0.0562 \text{ K/W}$$

$$Q = \frac{T_1 - T_2}{R_1} = \frac{758 - 727}{0.0562} = \mathbf{551.6535\ W/m^2}$$

To find k_2

$$Q = \frac{T_2 - T_3}{R_2}$$

$$551.65 = \frac{727 - 68.5}{R_2}$$

$$R_2 = 1.1937 \text{ K/W}$$

$$R_2 = \frac{L_2}{k_2 A_2}$$

$$k_2 = \frac{L_2}{R_2 A_2} = \frac{0.127}{1.1937 \times 1} = \mathbf{0.1064\ W/m\ K}$$

To find k_3

$$Q = \frac{T_3 - T_4}{R_3}$$

$$551.65 = \frac{68.5 - 68.48}{R_3}$$

$$R_3 = 3.625 \times 10^{-4}$$

$$R_3 = \frac{L_3}{k_3 A_3}$$

$$k_3 = \frac{L_3}{R_3 A_3} = \frac{0.00635}{3.625 \times 10^{-4} \times 1} = 175.15\ \text{W/m K}$$

So $k_3 = k$ for steel $=$ **175.15 W/m K.**

To find h_a

$$Q_{\text{conduction}} = Q_{\text{convection from gas side to fire brick}}$$

$$= 551.65 \text{ W}$$

$$Q_{\text{convection}} = h_a A (T_a - T_1)$$

$$551.65 = h_a \times 1 \times (760 - 758)$$

$$h_a = \frac{551.65}{760 - 758}$$

$$\mathbf{h_a = 275.83 \text{ W/m}^2\text{°C.}}$$

To find h_b

$$Q_{\text{convection}} \text{ from -steel plate to air} = h_b \times A \times (T_4 - T_b)$$

$$551.65 = h_b \times 1 \times (68.48 - 26.5)$$

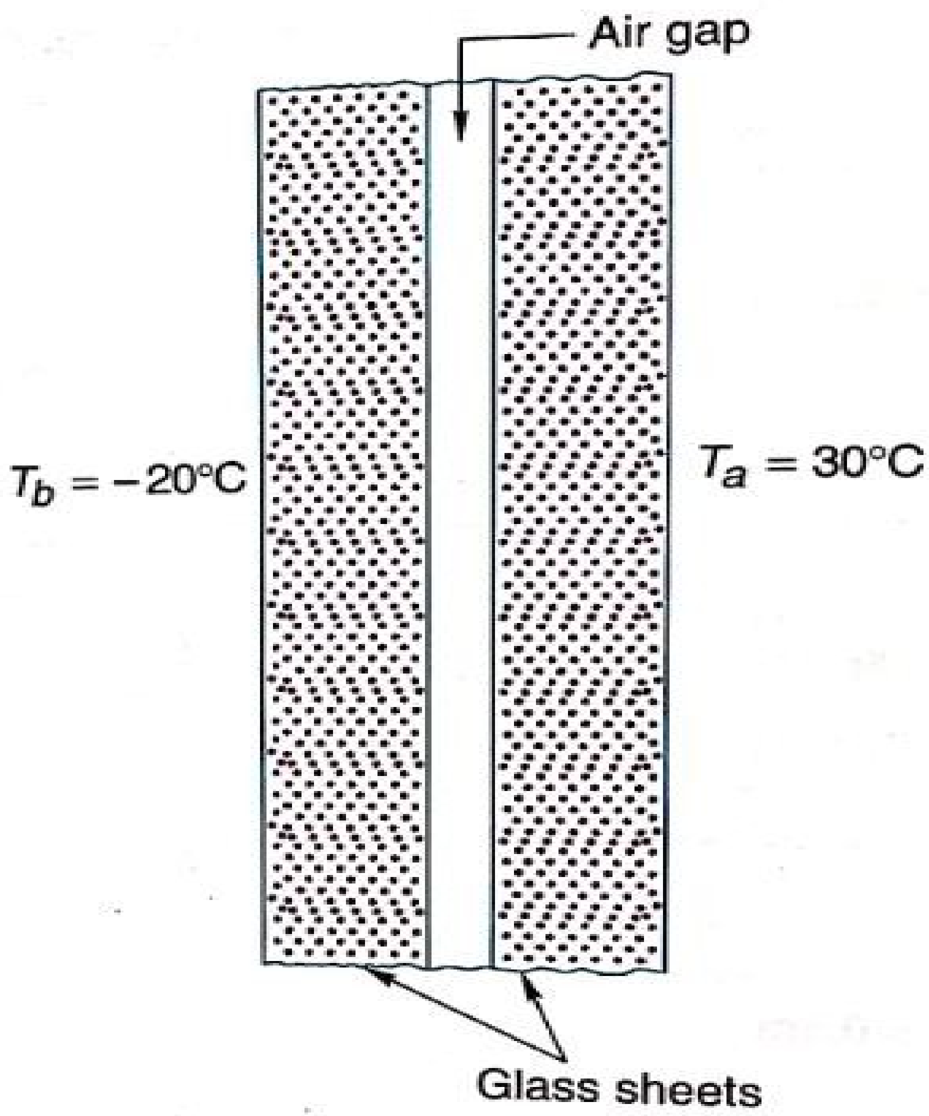
$$\mathbf{h_b = 13.141 \text{ W/m}^2\text{°C.}}$$

Example 3.9

The door of a cold storage plant is made from two 6 mm thick glass sheets separated by a uniform air of 2 mm. The temperature of the air inside the room is –20°C and the ambient air temperature is 3 Assuming the heat transfer coefficient between glass and air to be $23.26\ \mathrm{W/m^2K}$, determine the ra the heat leaking into the room per unit area of the door. Neglect convection effects in the air gap.

$$k_{glass} = 0.75\ \mathrm{W/mK}$$

$$k_{air} = 0.02\ \mathrm{W/mK}$$



Air gap
$T_b = -20°C$
$T_a = 30°C$
Glass sheets

Solution.

Referring to Eq. (3.32)

$$\frac{Q}{A}=\frac{T_a-T_b}{A(\sum R)}=\frac{T_a-T_b}{\dfrac{1}{h_a}+\dfrac{L_1}{k_1}+\dfrac{L_2}{k_2}+\dfrac{L_3}{k_3}+\dfrac{1}{h_b}}$$

$$=\frac{30-(-20)}{\dfrac{1}{23.26}+\dfrac{0.006}{0.75}+\dfrac{0.002}{0.02}+\dfrac{0.006}{0.75}+\dfrac{1}{23.26}}=\frac{50}{0.202}$$

$$= 247.5\ \text{W/m}^2$$

Problem 1.5: A composite wall is made of a 2.5 cm copper plate ($k = 355$ W/mK), a 3.2 mm layer of asbestos ($k = 0.110$ W/mK) and a 5 cm layer of fiber plate ($k = 0.049$ W/mK). The wall is subjected to an overall temperature difference of 560°C on the Cu plate side and 0°C on the fiber plate side. Estimate the heat flux through this composite wall and interface temperature between asbestos and fiber plate. (FAQ)

Given:

Thickness of Cu plate	$L_1 = 2.5$ cm $= 0.025$ m
Thickness of asbestos	$L_2 = 3.2$ mm $= 0.0032$ m
Thickness of fiber plate	$L_3 = 5$ cm $= 0.05$ m
Thermal conductivity of Cu plate	$k_1 = 355$ W/mK
Thermal conductivity of asbestos	$k_2 = 0.110$ W/mK
Thermal conductivity of fiber plate	$k_3 = 0.49$ W/mK
Overall temperature difference,	$\Delta T = 560°C$

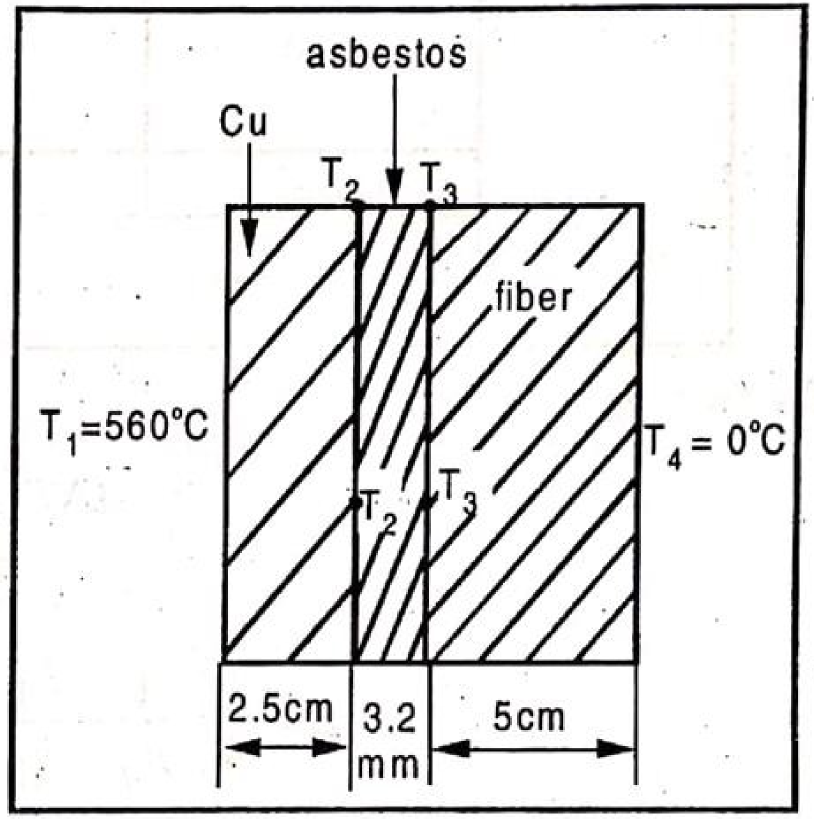
From HMT DB **page 44.**

Heat flux

$$\frac{Q}{A} = \frac{\Delta T}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}}$$

$$= \frac{560}{\frac{0.025}{355} + \frac{0.0032}{0.110} + \frac{0.05}{0.049}}$$

$$= 533.55 \text{ W/m}^3$$



$$\frac{Q}{A} = \frac{T_1 - T_2}{\frac{L_1}{k_1}} = \frac{T_2 - T_3}{\frac{L_2}{k_2}} = \frac{T_3 - T_4}{\frac{L_3}{k_3}}$$

$$533.55 = \frac{T_3 - 0}{\left(\frac{0.05}{0.049}\right)}$$

Interface temperature between asbestos and fiber plate

T_3 **= 544.43°C**

Problem 1.9.: *A composite wall is made up of three layers 15 cm, 10 cm and 12 cm of thickness. The first layer is made up of material with* $k = 1.45\ \text{W/m°C}$ *for 60% of area and rest of the material with* $k = 2.5\ \frac{\text{W}}{\text{m°C}}$. *The second layer is made with material of* $k = 12.5\ \text{W/m°C}$ *for 50% of the area and the rest of the material with* $k = 18.5\ \text{W/m°C}$. *The third layer is of single material with* $k = 0.76\ \text{W/m°C}$. *The composite slab is exposed to warm air at 26°C and cold air of −20°C on the other side. The inner and outer heat transfer coefficients are* $15\ \text{W/m}^2\text{°C}$ *and* $20\ \text{W/m}^2\text{°C}$. *Determine heat flux rate and interface temperatures.* *(FAQ)*

Solution

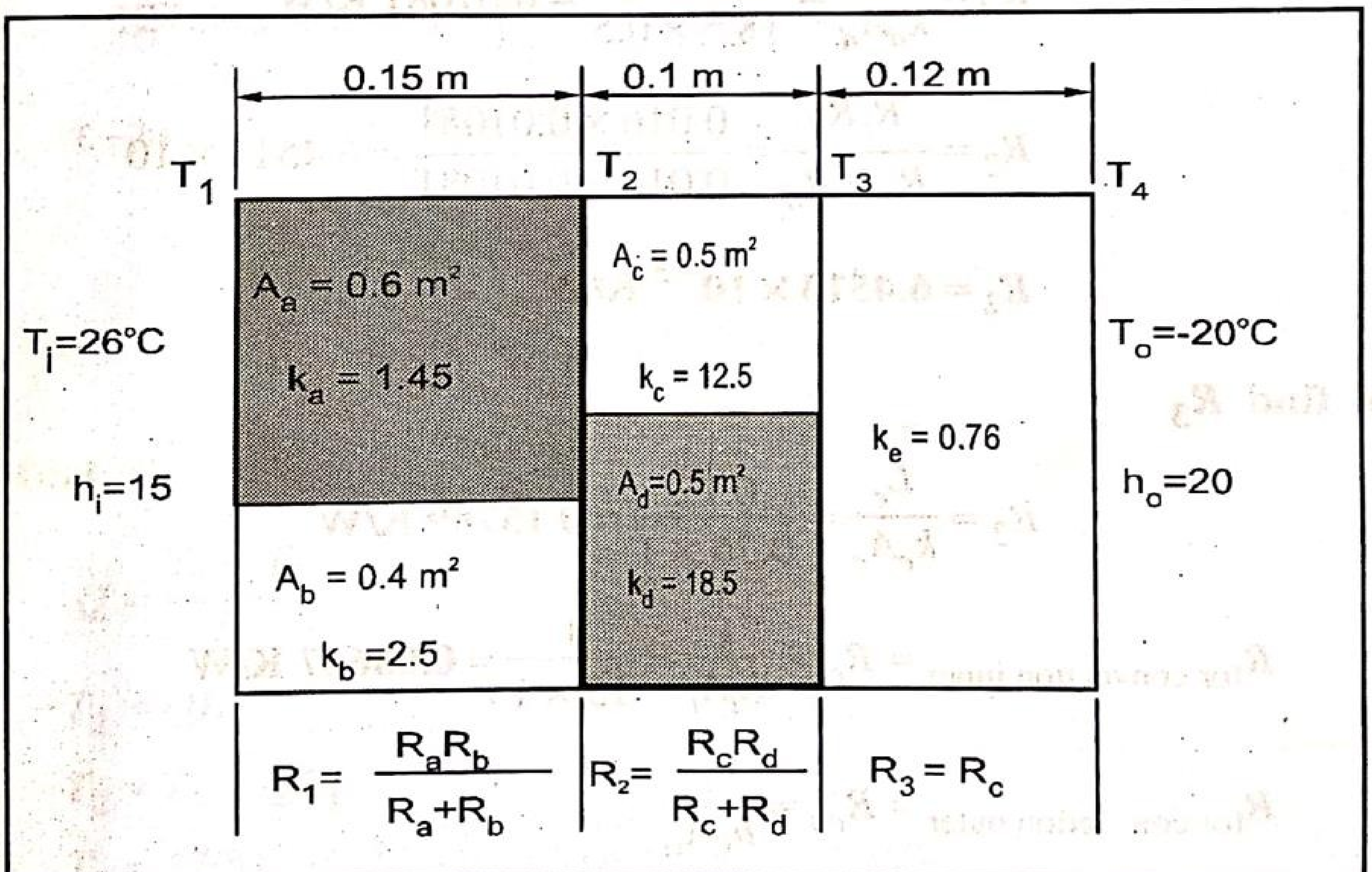
$$R = \frac{L}{KA}, \quad R_c' = \frac{1}{hA}$$

$$q = \frac{Q}{A} \quad Q = \frac{\Delta T}{R}$$

$$R_{eq} = \text{Equivalent } R$$

$$\frac{1}{R_{eq}} = \frac{1}{R_a} + \frac{1}{R_b}$$

$$\frac{1}{R_{eq}} = \frac{R_a + R_b}{R_a R_b} \text{ or } R_{eq} = \frac{R_a R_b}{R_a + R_b}$$



Assume $A = 1\ m^2$ (Since surface area is not given).

Refer **Pg 47** of CPK - Data book

To find R_1

$$R_a = \frac{L_a}{k_a A_a} = \frac{0.15}{1.45 \times 0.6} = 0.17241\ K/W$$

$$R_b = \frac{L_b}{k_b A_b} = \frac{0.15}{2.5 \times 0.4} = 0.15\ K/W.$$

$$R_1 = \frac{R_a R_b}{R_a + R_b} = \frac{0.17241 \times 0.15}{0.17241 + 0.15} = 0.080213\ K/W$$

$$\mathbf{R_1 = 0.080213\ K/W.}$$

To find R_2 $R_c = \frac{L_c}{k_c A_c} = \frac{0.1}{12.5 \times 0.5} = 0.016 \text{ K/W}$

$$R_d = \frac{L_d}{k_d A_d} = \frac{0.1}{18.5 \times 0.5} = 0.01081 \text{ K/W}$$

$$R_2 = \frac{R_c R_d}{R_c + R_d} = \frac{0.016 \times 0.01081}{0.016 + 0.01081} = 6.4513 \times 10^{-3}$$

$$\mathbf{R_2 = 6.4513 \times 10^{-3} \text{ K/W}}$$

To find R_3

$$R_3 = \frac{L_e}{k_e A_e} = \frac{0.12}{0.76 \times 1} = \mathbf{0.15789 \text{ K/W}}$$

$$R_{\text{for convection inner}} = R_{ci} = \frac{1}{h_i A_i} = \frac{1}{15 \times 1} = \mathbf{0.06667 \text{ K/W}}$$

$$R_{\text{for convection outer}} = R_{co} = \frac{1}{h_0 A_0}$$

$$= \frac{1}{20 \times 1} = \mathbf{0.05 \text{ K/W}}$$

To find Q

$$R = R_{ci} + R_1 + R_2 + R_3 + R_{co}$$

$$= 0.06667 + 0.080213 + 6.4513 \times 10^{-3} \ldots$$

$$= \mathbf{0.36122\ K/W.}$$

$$\frac{Q}{A} = q = \frac{(\Delta T)_{\text{overall}}}{R} = \frac{T_i - \ldots}{\ldots}$$

$$= \frac{26 - (- \ldots}{0 \ldots}$$

To find Interface Temperatures

To find T_1

$Q_{conducted} = Q_{convected}$ under steady state condition.

$$Q_{convection} = 127.345 = h_i A (T_i - T_1)$$

$$127.345 = 15 \times 1 \times (26 - T_1)$$

$$26 - T_1 = \frac{127.345}{15} = 8.4896$$

$$T_1 = 26 - 8.4896 = 17.51°C$$

$$\boxed{T_1 = 17.51°C.}$$

To find T_2

$$Q = \frac{T_1 - T_2}{R_1}$$

$$T_1 - T_2 = QR_1$$

$$T_2 = T_1 - QR_1$$

$$T_2 = 17.51 - (127.345 \times 0.080213) = 7.295°C$$

$$\boxed{T_2 = 7.295°C.}$$

To find T_3

$$Q = \frac{T_2 - T_3}{R_2}$$

$$T_2 - T_3 = QR_2$$

$$T_3 = T_2 - QR_2$$

$$T_3 = 7.295 - (127.345 \times 6.4513 \times 10^{-3}) = 6.4737°C$$

$$\boxed{T_3 = 6.4737°C.}$$

To find T_4

$$Q_{convected} = 127.345 = h_0 A\,(T_4 - T_0)$$

$$127.345 = 20 \times 1 \times (T_4 - (-20))$$

$$T_4 + 20 = \frac{127.345}{20} = 6.36725$$

$$\boxed{T_4 = -13.6327°C}$$

1.5 GENERAL DIFFERENTIAL EQUATION OF HE
CONDUCTION - CYLINDRICAL COORDIN

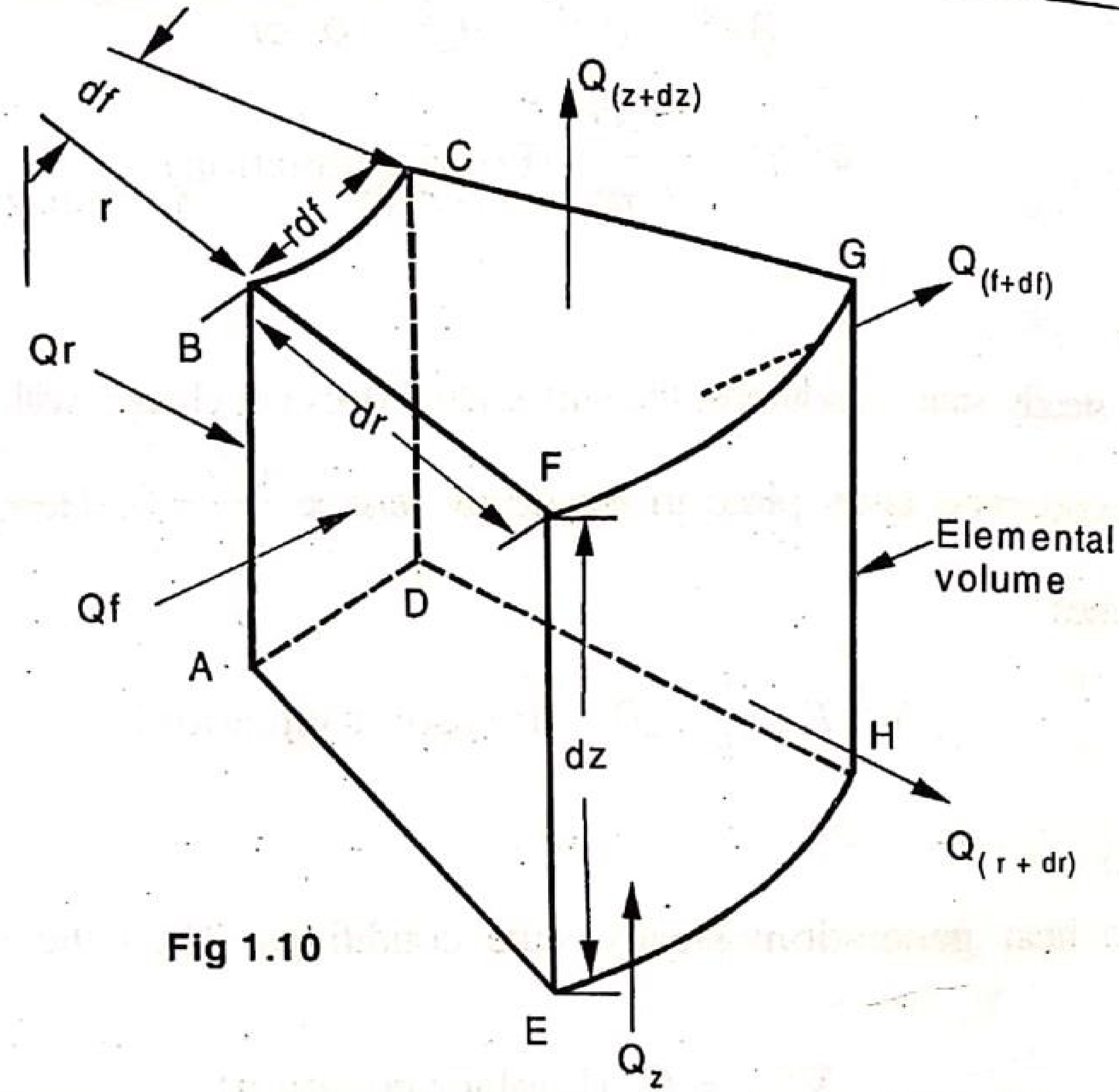
The heat conduction equation in cartes

for rectangular solids like slabs, cu

like rods and pipes, it is

1.10 shows a

equati



df
r
rdf
C
$Q_{(z+dz)}$
G
$Q_{(f+df)}$
Qr
B
dr
F
Elemental volume
Qf
D
A
H
dz
$Q_{(r+dr)}$
E
Q_z

Fig 1.10

Q_r = Heat conducted to the element in the 'r' direction through left face $ABCD$

Q_g = Heat generated with in the element

$\frac{dh}{dt}$ = Change in ethalpy per unit time

Q_{r+dr} = Heat conducted out of the element in 'r' direction through the right face $EFGH$

By applying I law of thermodynamics,

$$Q_r + Q_g = \frac{dh}{dt} + Q_{r+dr} \qquad ...(1.17)$$

$$Q_r = q_r A = -k A \frac{\partial T}{\partial r} = -k (r \cdot d\phi \cdot dz) \frac{\partial T}{\partial r} \qquad ...(1.18)$$

Where A = area of element $= r \cdot d\phi \cdot dz$

$$Q_g = q_g (dr \cdot r\, d\phi \cdot dz) \qquad ...(1.19)$$

$\frac{dh}{dt}$ = mass of the element × specific heat × change in temperature of the element in time dt

$$\frac{dh}{dt} = [\rho\, (dr \cdot r d\,\phi \cdot dz)] \times c_p \times \frac{\partial T}{\partial t} \qquad ...(1.20)$$

$$Q_{r+dr} = q_r A + \left[\frac{\partial}{\partial r} (q_r A) \right] dr = -kA \frac{\partial T}{\partial r} + \frac{\partial}{\partial r} \left(-kA \frac{\partial T}{\partial r} \right) dr \qquad ...(1.21)$$

where

k = thermal conductivity of the material in the $r-$direction

$\partial T/\partial r$ = temperature gradient in the $r-$direction

q_r = heat flux in the $r-$direction at r,

i.e. at left face, i.e. at $ABCD$ (W/m^2)

q_g = internal energy generated per unit time and per unit volume W/m^3

ρ = density of the material (kg/m^3)

$(\partial T/\partial r)\,dr$ = change in temperature through distance dr

$Q_r + Q_g = \frac{dh}{dt} + Q_{r+dr}$

Substituting Eqs. (1.18), (1.9), (1.20) and (1.21) in Eq. (1.17) we get

$$-kA\frac{\partial T}{\partial r} + q_g\,A dr = \rho\, c_p\, A dr \frac{\partial T}{\partial t} - \left[kA\frac{\partial T}{\partial r} + \frac{\partial}{\partial r}\left(kA\frac{\partial T}{\partial r}\right)dr\right]$$

R.A.dr.Cp

A

$$k\,(d\phi \cdot dr \cdot dz)\,\frac{\partial}{\partial r}\left(r\,\frac{\partial T}{\partial r}\right) + q_g\,(r \cdot d\phi \cdot dz \cdot dr) = \rho \cdot c_p\, r \cdot dz \cdot dr \cdot d\phi\,\frac{\partial T}{\partial t}$$

$$k\left[r\,\frac{\partial^2 T}{\partial r^2} + \frac{\partial T}{\partial r}\right] + q_g\, r = \rho \cdot c_p\, r\,\frac{\partial T}{\partial t}$$

$$\left.\begin{array}{l} \left[\dfrac{\partial^2 T}{\partial r^2} + \dfrac{1}{r}\,\dfrac{\partial T}{\partial r}\right] + \dfrac{q_g}{k} = \dfrac{\rho\, c_p}{k}\,\dfrac{\partial T}{\partial t} \\ \dfrac{1}{r}\,\dfrac{\partial}{\partial r}\left(r\,\dfrac{\partial T}{\partial r}\right) + \dfrac{q_g}{k} = \dfrac{\rho\, c_p}{k}\,\dfrac{\partial T}{\partial t} \end{array}\right\} \quad \ldots(1.22)$$

Equation (1.22) is the one-dimensional cylindrical coordinate time-dependent equation for heat conduction with internal heat generation.

This Equation (1.22) can be reduced to different cases as follows:

Case 1: Steady state, one-dimensional heat transfer **with** internal heat generation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{q_g}{k}=0 \qquad \text{...(1.22 (a))}$$

Case 2: Steady state, one-dimensional, **without** internal heat generation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=0 \qquad \text{...(1.22 (b))}$$

Case 3: Unsteady state, one-dimensional, without heat generation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)=\frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad \text{...(1.22(c))}$$

The three dimensional general heat conduction equation in cylindrical coordinates is given as

$$\left[\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}+\frac{\partial^2 T}{\partial z^2}\right]+\frac{q_g}{k}=\frac{1}{\alpha}\cdot\frac{\partial T}{\partial t} \qquad \text{...(1.22(d))}$$

1.8 HEAT CONDUCTION THROUGH A HOLLOW CYLINDER

Figure 1.16 shows a long hollow cylinder made of a material having constant thermal conductivity and insulated at both ends. The inner and outer radii are r_1 and r_2, respectively. The length of the cylinder is L.

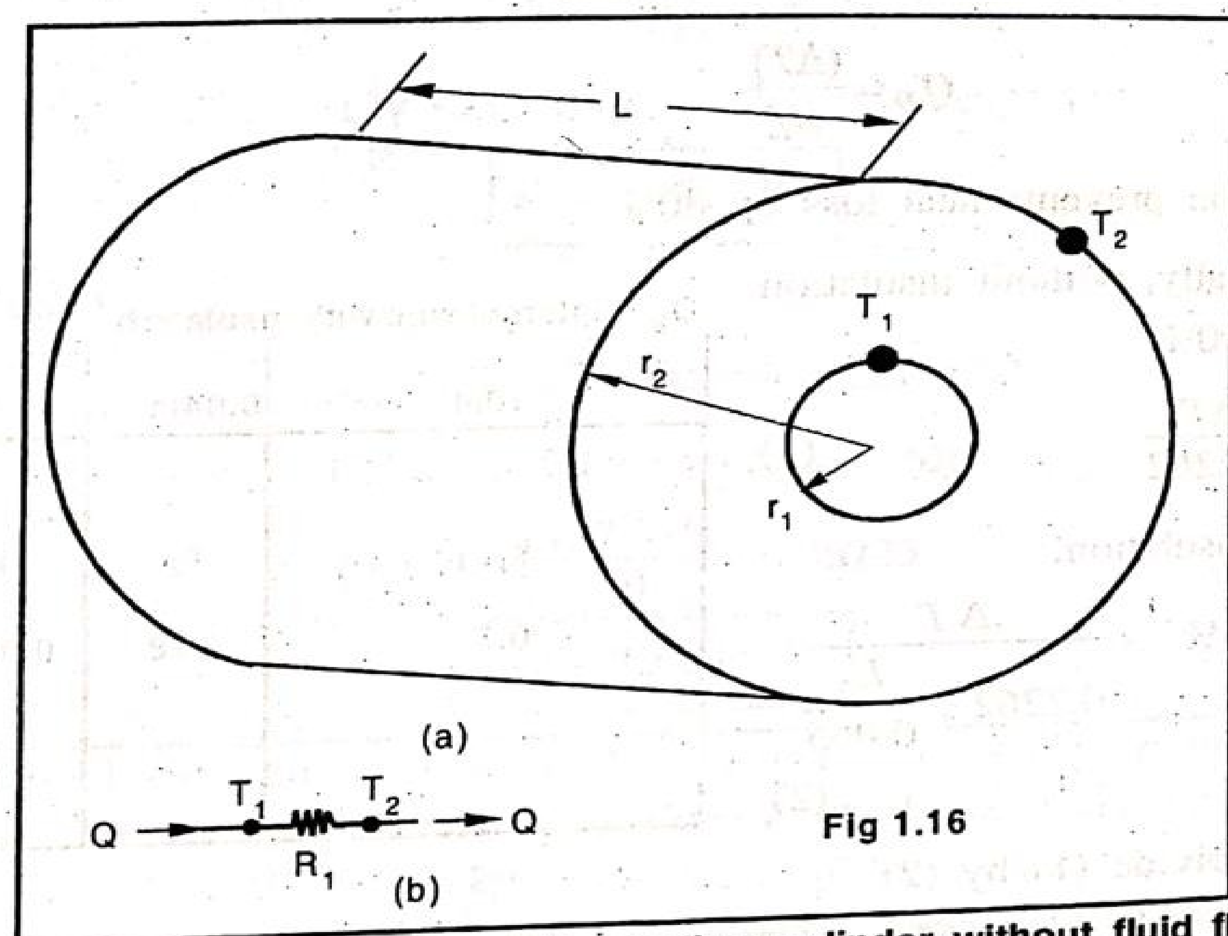


Fig. 1.16 (a) Conduction through a hollow cylinder without fluid flowing inside and outside the cylinder
(b) Equivalent thermal resistance circuit

... conduction equation in cylindrical coordinates is

(b) Equivalent thermal resis...

The general heat conduction equation in cylindrical coordinates is given by

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}+\frac{\partial^2 T}{\partial z^2}+\frac{q_g}{k}=\frac{\partial T}{\alpha \partial t} \quad \text{...(1.49)}$$

Assumptions:

Steady state: $\frac{\partial T}{\partial t}=0$

No heat generation: $q_g=0$

One dimension: $\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} = \frac{\partial^2 T}{\partial z^2} = 0$

Substitute these assumptions in Eq. (1.49), we have

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = 0; \frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

$$\therefore \frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0, \text{ since } \frac{1}{r} \neq 0 \qquad \text{...(1.50)}$$

Integrating Eq. (1.50) twice we get

$$r\frac{dT}{dr} = C_1 \text{ or } \frac{dT}{dr} = \frac{C_1}{r}$$

$$T = ln\, r\, C_1 + C_2 \qquad \text{...(1.51)}$$

where C_1 and C_2 are arbitrary constants

undary conditions

At $r = r_1, T = T_1$

At $r = r_2, T = T_2$

Substituting these boundary conditions in Eq. (1.51)

$$T_1 = ln\, r_1\, C_1 + C_2$$

$$T_2 = ln\, r_2\, C_1 + C_2$$

Solving the above two equations, we have

$$C_1 = \frac{T_1 - T_2}{ln \frac{r_1}{r_2}} = \frac{T_2 - T_1}{ln \frac{r_2}{r_1}}$$

$$C_2 = T_1 - C_1\, ln\, r_1 = T_1 - \frac{(T_2 - T_1)\, ln\, r_1}{ln \frac{r_2}{r_1}}$$

Substituting C_1 and C_2 in Eq. (1.51), we get

$$T = \ln r \left[\frac{T_2 - T_1}{\ln \frac{r_2}{r_1}} \right] + T_1 - (T_2 - T_1) \left[\frac{\ln r_1}{\ln \frac{r_2}{r_1}} \right]$$

$$T = \frac{T_2 - T_1}{\ln \frac{r_2}{r_1}} (\ln r - \ln r_1) + T_1$$

$$\frac{T - T_1}{T_2 - T_1} = \frac{\ln \frac{r}{r_1}}{\ln \frac{r_2}{r_1}} \qquad \text{...(1.52)}$$

Equation (1.52) gives the temperature distribution in a hollow cylinder. The heat flow rate through the cylinder over the surface area A is given by Fourier's conduction equation.

$$Q = -kA \left| \frac{dT}{dr} \right| \text{ when } (r = r_1)$$

Substituting dT/dr from Eq. (1.51) into the above equation

(when $r = r_1$)

$$\therefore \quad Q = -kA \frac{(T_2 - T_1)}{ln \frac{r_2}{r_1}} \times \frac{1}{r_1} = \frac{k2\pi r_1 L (T_1 - T_2)}{ln \frac{r_2}{r_1}} \times \frac{1}{r_1}$$

$$= \frac{2\pi k L (T_1 - T_2)}{ln \frac{r_2}{r_1}} \qquad \text{...(1.53)} \quad [A = 2\pi r L]$$

$$\text{or } Q = \frac{T_1 - T_2}{\left(\frac{ln \frac{r_2}{r_1}}{2\pi k L} \right)} = \frac{T_1 - T_2}{R_{th}} \qquad \text{...(1.54)}$$

$\therefore R_{th} =$ Thermal resistance for conduction heat transfer

.9 HEAT CONDUCTION THROUGH COMPOSITE (COAXIAL) CYLINDERS WITH CONVECTION

Consider the rate of heat transfer through a composite cylinder as ıown in **Figure 1.17 (a)** and its equivalent thermal resistance in **Figure .17 (b)**

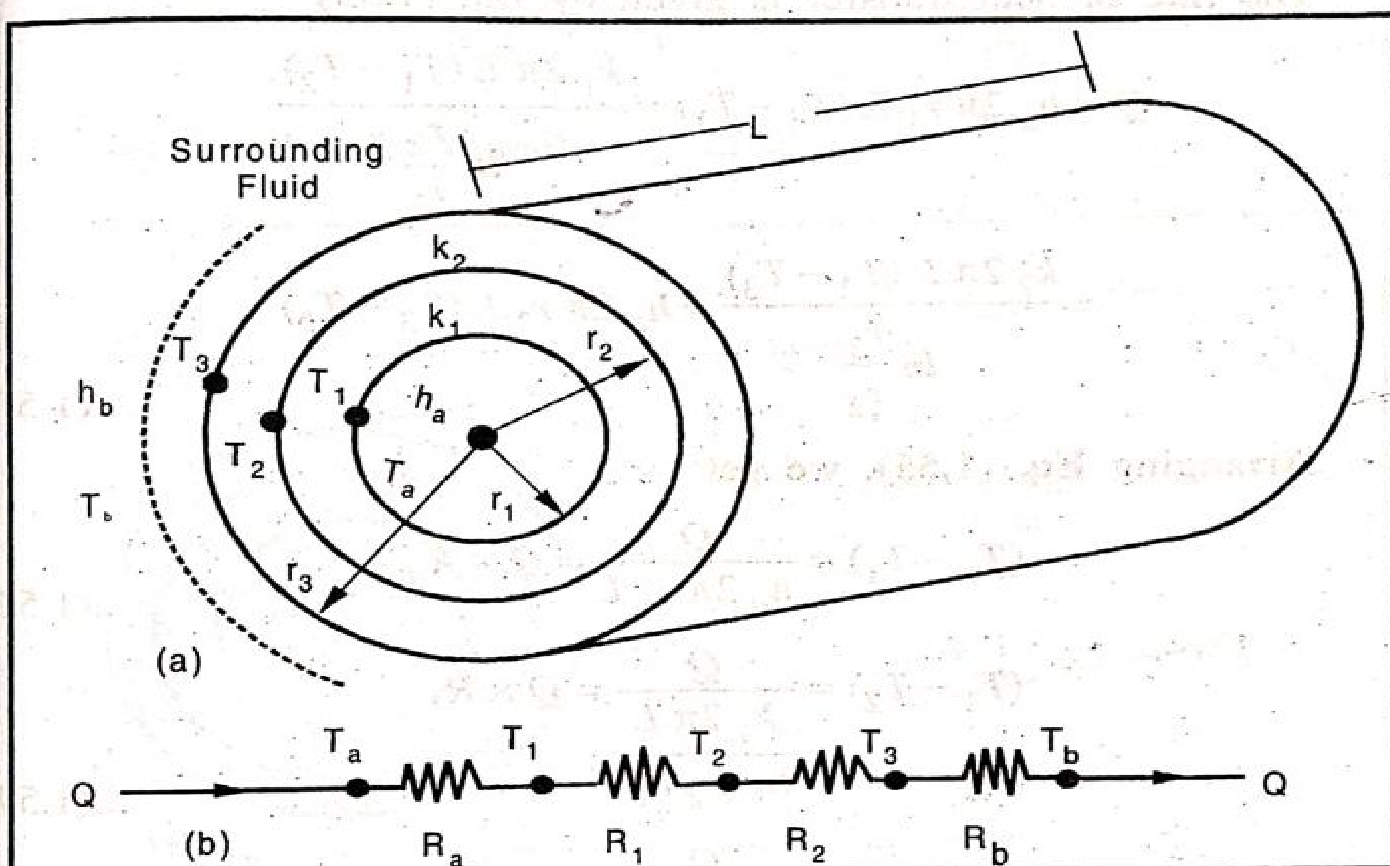


Fig. 1.17 (a) Conduction through a composite cylinder with fluid flowing inside and outside the cylinder
(b) Equivalent thermal resistance circuit

T_1, T_2, T_3 = Temperature at inlet surface, between first and second cylinders and outer surfac

L = Length of the cylinde

h_a, h_b = Conve

T_a, T_b = Temperature of the fluid flowing inside and outside the composite cylinder

k_1, k_2 = Thermal conductivity of the first and second material, respectively

The rate of heat transfer is given by Eq. (1.53)

$$Q = h_a \, 2\pi \, r_1 \, L \, (T_a - T_1) = \frac{k_1 \, 2\pi \, L \, (T_1 - T_2)}{ln \dfrac{r_2}{r_1}}$$

$$= \frac{k_2 \, 2\pi \, L \, (T_2 - T_3)}{ln \dfrac{r_3}{r_2}} = h_b \, 2\pi \, r_3 \, L \, (T_3 - T_b) \qquad ...(1.55)$$

Arranging **Eq. (1.55)**, we get

$$(T_a - T_1) = \frac{Q}{h_a\, 2\pi\, r_1\, L} = Q \times R_a \quad \text{...(1.56)}$$

$$(T_1 - T_2) = \frac{Q}{\dfrac{k_1\, 2\pi\, L}{ln\,(r_2/r_1)}} = Q \times R_1 \quad \text{...(1.57)}$$

$$(T_2 - T_3) = \frac{Q}{\dfrac{k_2\, 2\pi\, L}{ln\,(r_3/r_2)}} = Q \times R_2 \quad \text{...(1.58)}$$

$$(T_3 - T_b) = \frac{Q}{h_b\, 2\pi\, r_3\, L} = Q \times R_b \quad \text{...(1.59)}$$

Adding **Eqs.** from **1.56** to **1.59,** we get

$$(T_a - T_b) = Q\,(R_a + R_1 + R_2 + R_b) \quad \text{...(1.60)}$$

$$(T_a - T_b) = Q\left(\frac{1}{h_a\, 2\pi\, L\, r_1} + \frac{ln\,\frac{r_2}{r_1}}{k_1\, 2\pi\, L} + \frac{ln\,\frac{r_3}{r_2}}{L\, k_2\, 2\pi} + \frac{1}{h_b\, 2\pi\, L\, r_3}\right) \quad ...(1.61)$$

$$\text{or} \quad Q = \frac{(T_a - T_b)\, 2\pi\, L}{\frac{1}{h_a\, r_1} + \frac{ln\, r_2/r_1}{k_1} + \frac{ln\, r_3/r_2}{k_2} + \frac{1}{h_b\, r_3}} \quad ...(1.62)$$

9.1 Summary - Composite Cylinder

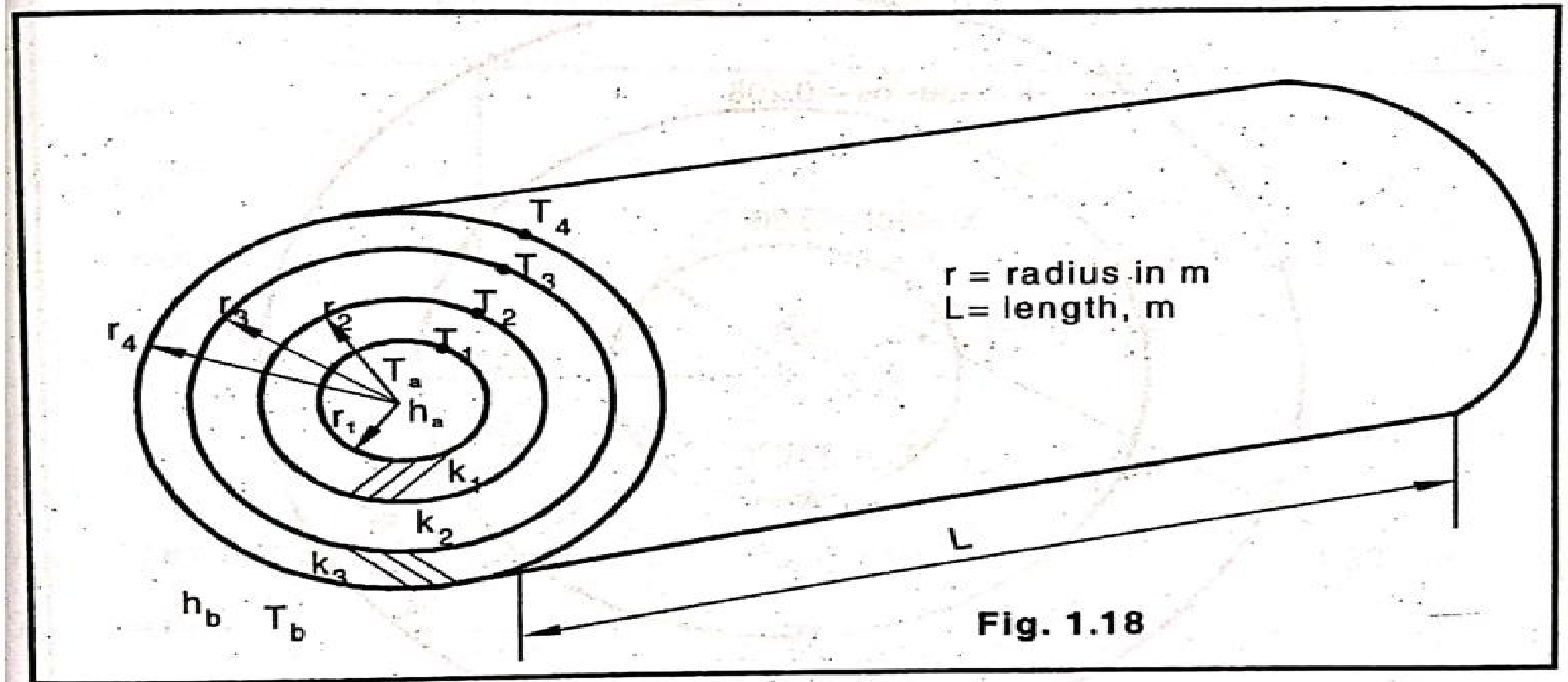


Fig. 1.18

Refer from **Pg 46** of HMT Data book - CPK.

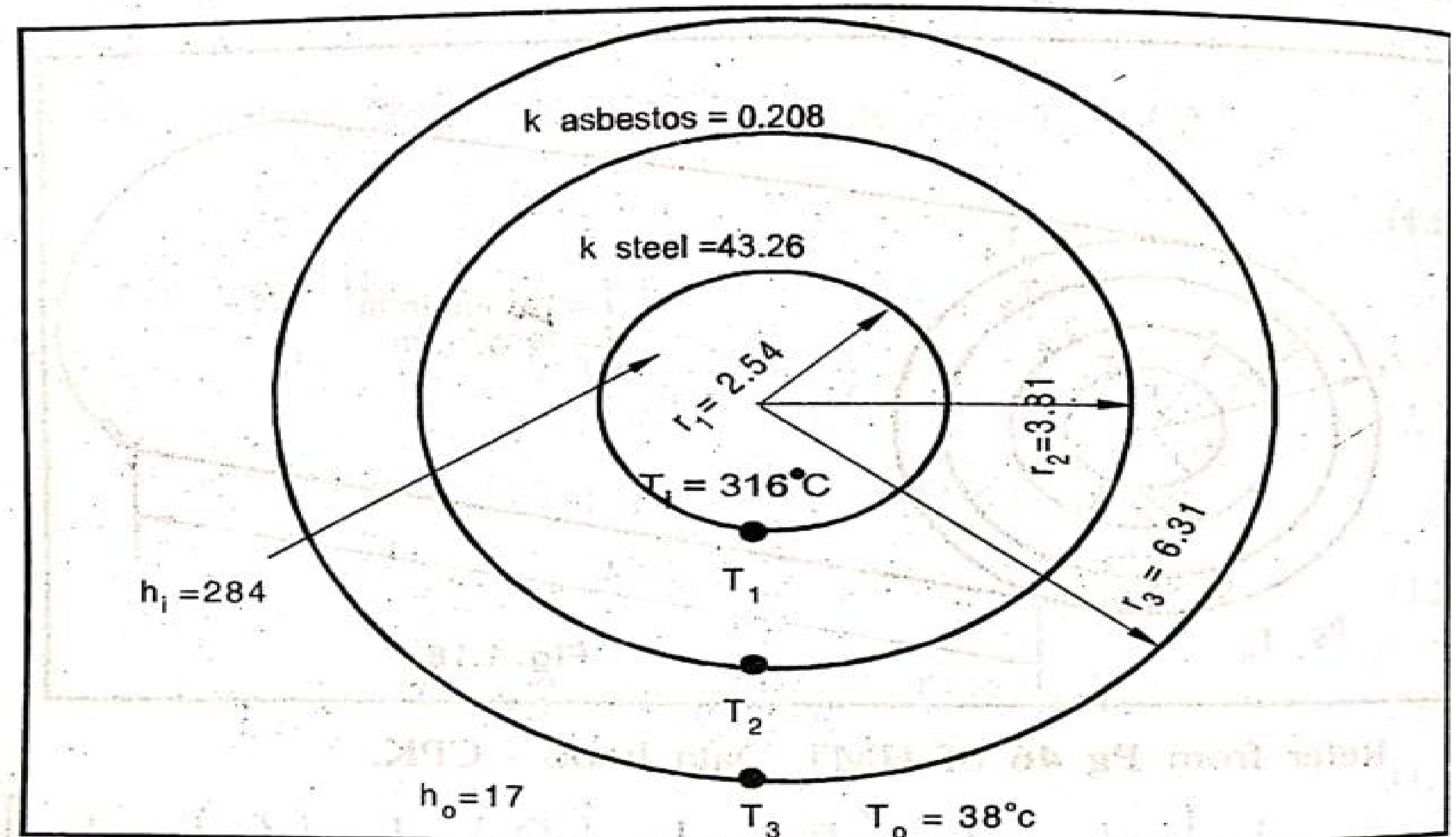
$$R = \frac{1}{2\pi L}\left[\frac{1}{h_a r_1} + \frac{1}{k_1}\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{k_2}\ln\left(\frac{r_3}{r_2}\right) + \frac{1}{k_3}\ln\left(\frac{r_4}{r_3}\right) + \frac{1}{h_b r_4}\right]$$

$$Q = \frac{(\Delta T)_{overall}}{R}$$

T_i or $T_a =$ Inner temperature; T_o or $T_b =$ Outer temperature

Problem 1.11: *A steel tube of 5.08 cm ID and 7.62 cm OD is cove[red] with 2.5 cm thick of asbest[os] {k_{steel} = 43.26 W/m°K ; $k_{asbestos}$ = 0.208 W/m°C. The inside surf[ace] receives heat from hot gases at 316°C with heat transfer coefficient [of] 284 W/m²°C whereas outer surface is exposed to air at 38°C with h[eat] transfer coefficient of 17 W/m²C. Determine (1) heat loss for 3 [m] length.* (FA[...]

Solution



$$R = \frac{1}{\ldots}\left[\frac{1}{\ldots} \ldots \left(\frac{r_2}{\ldots}\right) \ldots \left(\frac{r_3}{\ldots}\right) \ldots\right]$$

$$R = \frac{1}{2\pi L}\left[\frac{1}{h_i r_1} + \frac{1}{k_1} ln\left(\frac{r_2}{r_1}\right) + \frac{1}{k_2} ln\left(\frac{r_3}{r_2}\right) + \frac{1}{h_o r_3}\right]$$

$$= \frac{1}{2\pi \times 3}\left[\frac{1}{284 \times 0.0254} + \frac{1}{43.26} ln\left(\frac{0.0381}{0.0254}\right) + \frac{1}{0.208} ln\left[\frac{0.0631}{0.0381}\right] + \frac{1}{17 \times 0.0631}\right]$$

$$= 0.05305\ [0.1386 + 9.37 \times 10^{-3} + 2.426 + 0.9322]$$

$$= 0.18599 \text{ K/W}$$

$$Q = \frac{(\Delta T)_{overall}}{R} = \frac{316 - 38}{0.18599} = 1494.665 \text{ W}.$$

$$\mathbf{Q = 1494.665\ W}$$

Problem 1.12: *A steel tube of 50 mm ID and 80 mm OD is covered by 30 mm thick of asbestos. The thermal conductivity of steel, asbestos are* 45 W/m K, 0.2 W/m K. *The tube receives heat from hot gases at* 400°C *with heat transfer coefficient of* 300 W/m²°C. *The outer surface is exposed to air at* 30°C *with heat transfer coefficient of* 15 W/m² K. *Determine (1) heat loss /m length (2) Interface temperature and surface temperature.* *(FAQ)*

Solution

$r_1 = 0.025$ m

$r_2 = 0.04$ m

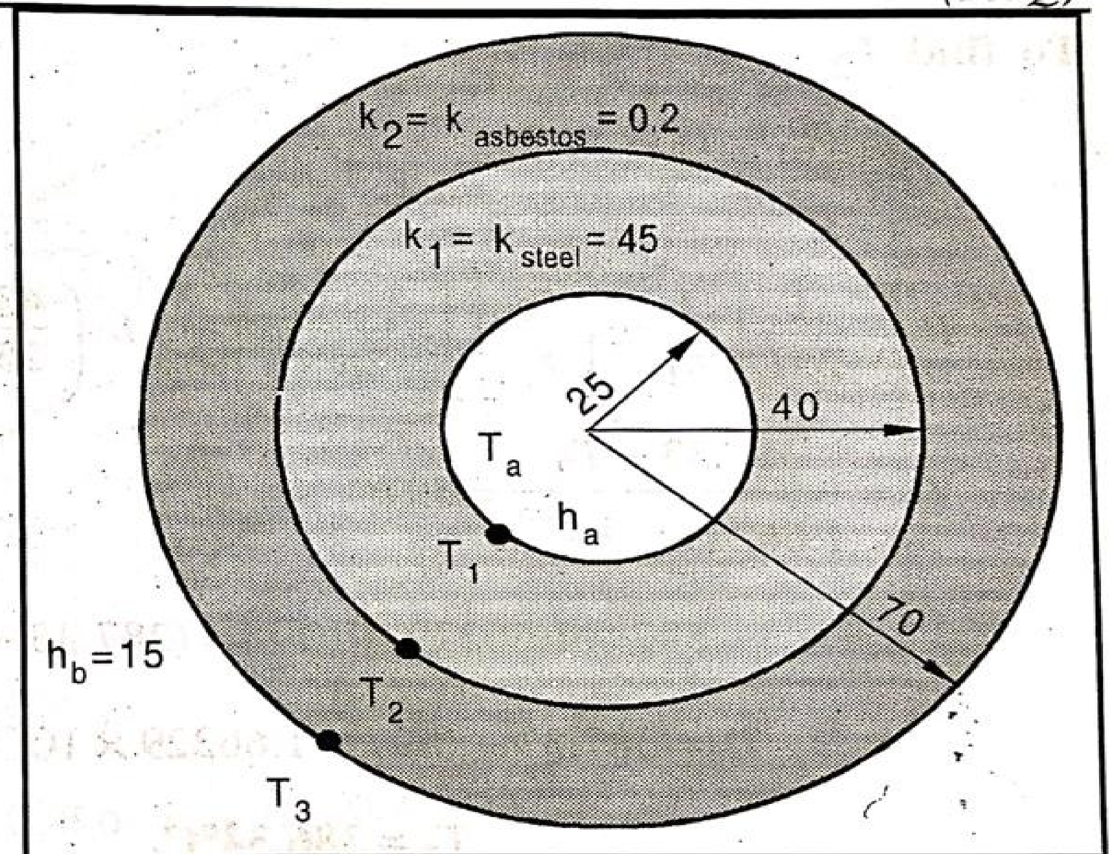
$r_3 = 0.07$ m

$T_{air} = T_b = 30°C$

$h_b = 15\ W/m^2°C$

$h_a = 300\ W/m^2°C$

$T_a = 400°C$



To find Q

$$R = \frac{1}{2\pi L}\left[\frac{1}{h_a r_1} + \frac{1}{k_1} ln\left(\frac{r_2}{r_1}\right) + \frac{1}{k_2} ln\left[\frac{r_3}{r_2}\right] + \frac{1}{h_b r_3}\right]$$

$$= \frac{1}{2\pi \times 1}\left[\frac{1}{300 \times 0.025} + \frac{1}{45} ln\left(\frac{40}{25}\right) + \frac{1}{0.2} ln\left(\frac{70}{40}\right) + \frac{1}{15 \times 0.07}\right]$$

$$= \frac{1}{2\pi}[\,3.894\,] = 0.6197 \text{ K/W}$$

$$Q = \frac{(\Delta T)_{overall}}{R} = \frac{T_a - T_b}{R} = \frac{(400 - 30)}{0.6197} = 597 \text{ W}$$

Q **= 597 Watts.**

To find Interface temperatures

To find T_1

$$Q = h_a A\,(T_a - T_1) = 300 \times (2\pi r_1 \times L)\,(400 - T_1)$$

$$597 = 47.12(400 - T_1)$$

$$T_1 = 400 - \left(\frac{597}{47.12}\right)$$

$$= 387.331°C$$

$$\boxed{T_1 = 387.33°C.}$$

To find T_2

$$Q = \frac{(T_1 - T_2)}{R_1}$$

$$R_1 = \frac{1}{2\pi L}\left[\frac{1}{k_1} ln\left[\frac{r_2}{r_1}\right]\right] = \frac{1}{2\pi \times 1}\left[\frac{1}{45} ln\left(\frac{40}{25}\right)\right] = 1.66229 \times 10^{-3}$$

$$Q = \frac{387.33 - T_2}{1.66229 \times 10^{-3}} = 597$$

$$597 \times 1.66229 \times 10^{-3} = (387.33 - T_2)$$

$$T_2 = 387.33 - (597 \times 1.66229 \times 10^{-3}) = 386.34°C$$

$$\mathbf{T_2 = 386.34°C.}$$

To find T_3

$$Q = h_b A\,(T_3 - T_b)$$

$$597 = 15 \times (2\pi r_3 L)\,(T_3 - 30)$$

$$597 = 15 \times (2\pi \times 0.07 \times 1)\,(T_3 - 30)$$

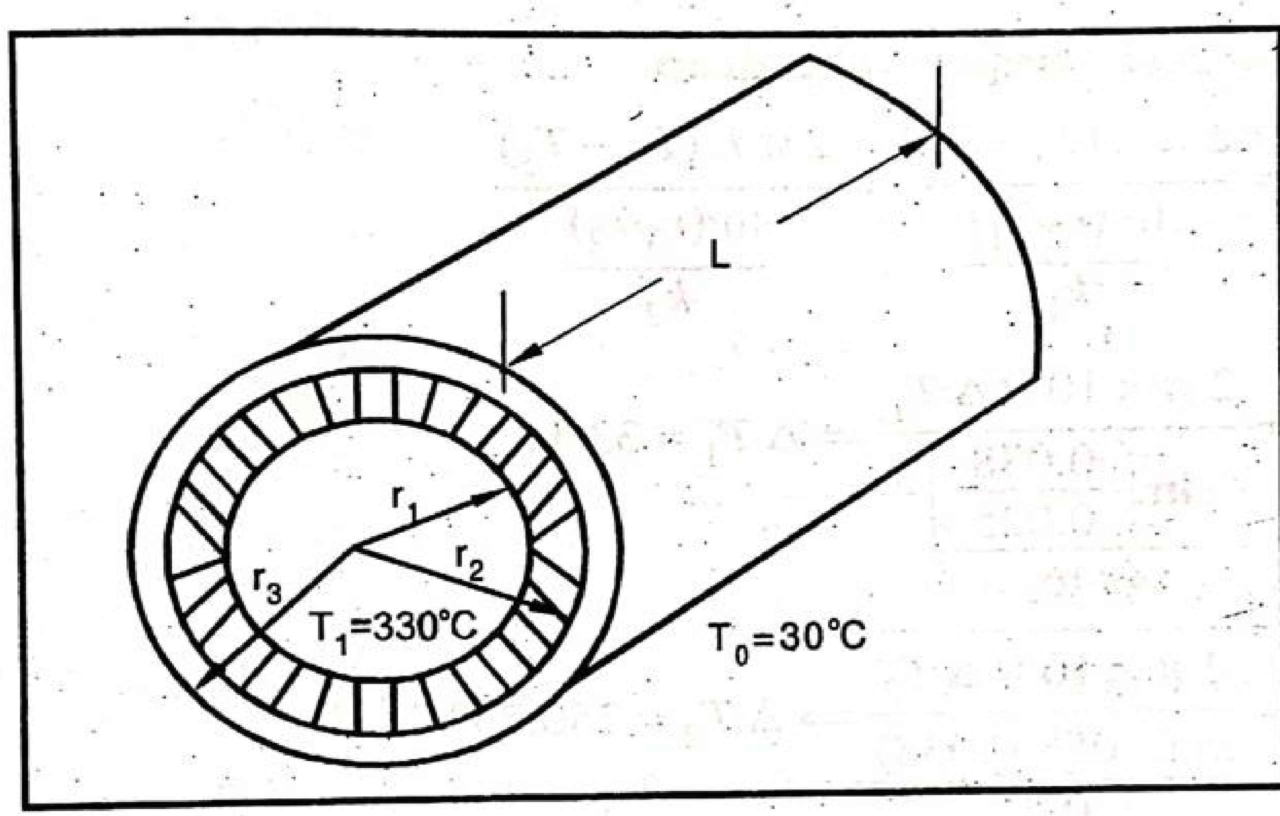
$$(T_3 - 30) = 90.49$$

$$T_3 = 90.49 + 30 = 120.491°C$$

$$\mathbf{T_3 = 120.491°C.}$$

Problem 1.13 *A steel tube of 5 cm ID, 7.6 cm OD and* $k = 15$ W/mK *is covered with an insulation of thickness 2 cm and thermal conductivity 0.2 W/m.K. A hot gas at* 330°C *and* $h = 400$ W/m^{2}K *flows inside the tube. The outer surface of the insulation is exposed to cold air at* 30°C *with* $h = 60$ W/m^2.K. *Assuming a tube lengbth of 10 m, find the heat loss from the tube to the air. Also find, across which layer the largest temperature drop occurs.* *(FAQ)*

Given



$r_1 = 2.5$ cm= 0.025 m, $k_1 = 15$ W/mK

$r_2 = 3.8$ cm= 0.038 m, $k_2 = 0.2$ W/mK

$r_3 = 0.038 + 0.02 = 0.058$ m

Inside temperature, $T_i = 330°C$

$h_i = 400$ W/m^{2}K

Outside temperature, $T_0 = 30°C$

$h_0 = 60$ W/m^{2}K

Tube length, $L = 10$ m

Heat loss from tube to air (HMT DB pg **No. 46**)

$$Q = \frac{2\pi L\,[T_i - T_0]}{\dfrac{1}{h_i\, r_1} + \dfrac{\ln (r_2/r_1)}{k_1} + \dfrac{\ln (r_3/r_2)}{k_2} + \dfrac{1}{h_0\, r_3}}$$

$$= \frac{2\pi \times 10\,(300)}{\dfrac{1}{400 \times 0.025} + \dfrac{\ln (0.038/0.025)}{15} + \dfrac{\ln (0.058/0.038)}{0.2} + \dfrac{1}{60 \times 0.058}}$$

$$= \frac{18849.56}{0.1 + 0.0279 + 2.114 + 0.287} = 7451.77 \text{ W}$$

To find largest temperature drop

$$Q = \frac{2\pi L\,(T_1 - T_2)}{\dfrac{\ln (r_2/r_1)}{k_1}} = \frac{2\pi L\,(T_2 - T_3)}{\dfrac{\ln (r_3/r_2)}{k_2}}$$

$$7451.77 = \frac{2\pi \times 10 \times \Delta T_1}{\dfrac{\ln\left(\dfrac{0.038}{0.025}\right)}{15}} \Rightarrow \Delta T_1 = 33°\text{C}$$

$$7451.77 = \frac{2\pi \times 10 \times \Delta T_2}{\dfrac{\ln (0.058/0.038)}{0.2}} \Rightarrow \Delta T_2 = 250.75°\text{C}$$

Largest temperature drop occurs in outer layer.

Transient Heat Conduction

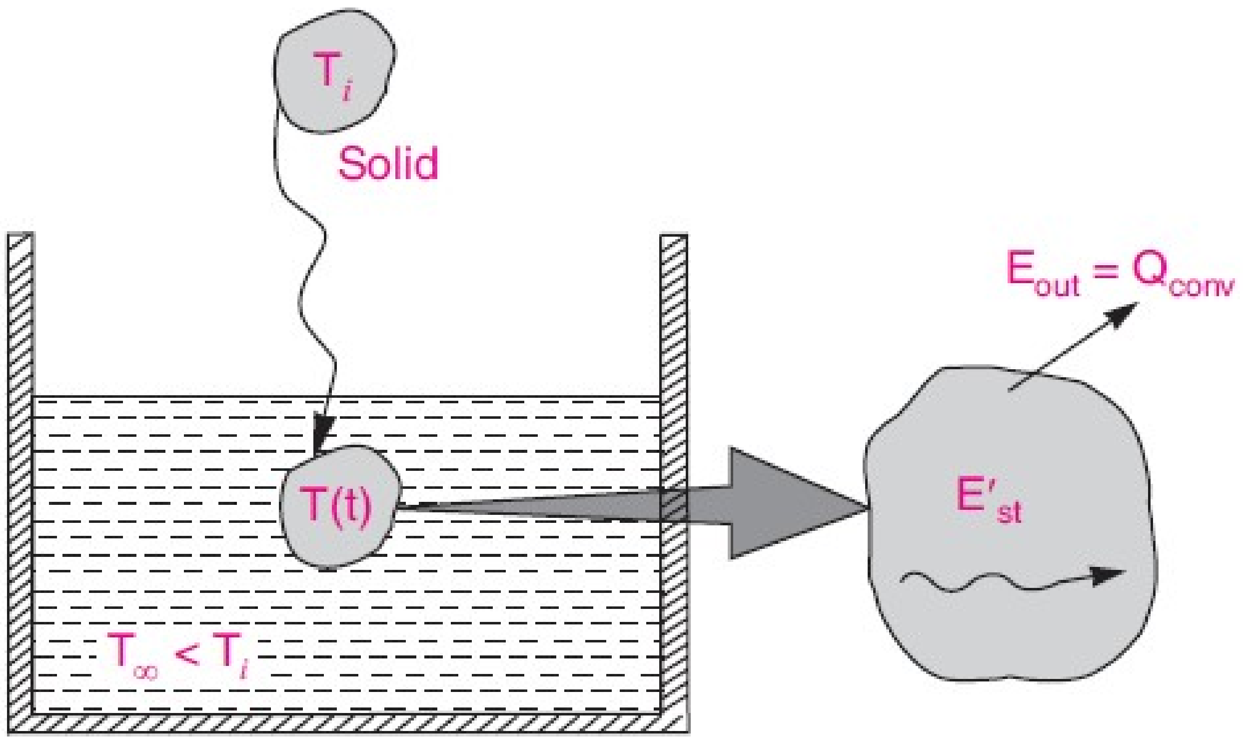


Fig. 6.1. Solid suddenly exposed to convection environment at T_∞

The initial temperature of solid T_i (Fig. 6.1) is greater than ambient fluid temperature, T_∞, the eqn. (6.1) leads to, When the heat energy is being added or removed to or from a body, its energy content (internal energy) changes, resulting into change in its temperature at each point within the body over the time. During this transient period, the temperature becomes function of time as well as direction in the body. The conduction occurred during this period is called *transient (unsteady state) conduction. Therefore,* in unsteady state

$$T = f(x, t)$$

= Function of direction and time

During transient heat conduction, the energy balance on a body yields to

The net rate of heat transfer with the body

= Net rate of internal energy change of the body.

6.1.1. Systems with Negligible Internal Resistance : Lumped System Analysis

If the physical size of the body is very small, the temperature gradient exists in the body is negligible. The small body can be assumed at uniform temperature throughout at any time. The analysis of the unsteady heat transfer with negligible temperature gradients is called the *lumped system analysis.*

Consider a solid of volume V, surface area A_s, thermal conductivity k, density ρ, specific heat C and initially at uniform temperature T_i is suddenly immersed in a well stirred fluid, kept at uniform temperature T_∞. The heat is dissipated by convection into a fluid from its surface, with convection coefficient h.

In absence of any temperature gradient in solid, the energy balance for element is :

The rate of heat flow out the solid through the boundary surface(s)

= The rate of decrease of internal energy of the solid

or $$hA_s(T - T_\infty) = -mC\frac{dT}{dt} \quad ...(6.1)$$

where, $m = \rho V$, mass of the body

and $T = f(t)$, a function of time.

or

or

or

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left\{-\frac{hA_s t}{\rho VC}\right\}$$

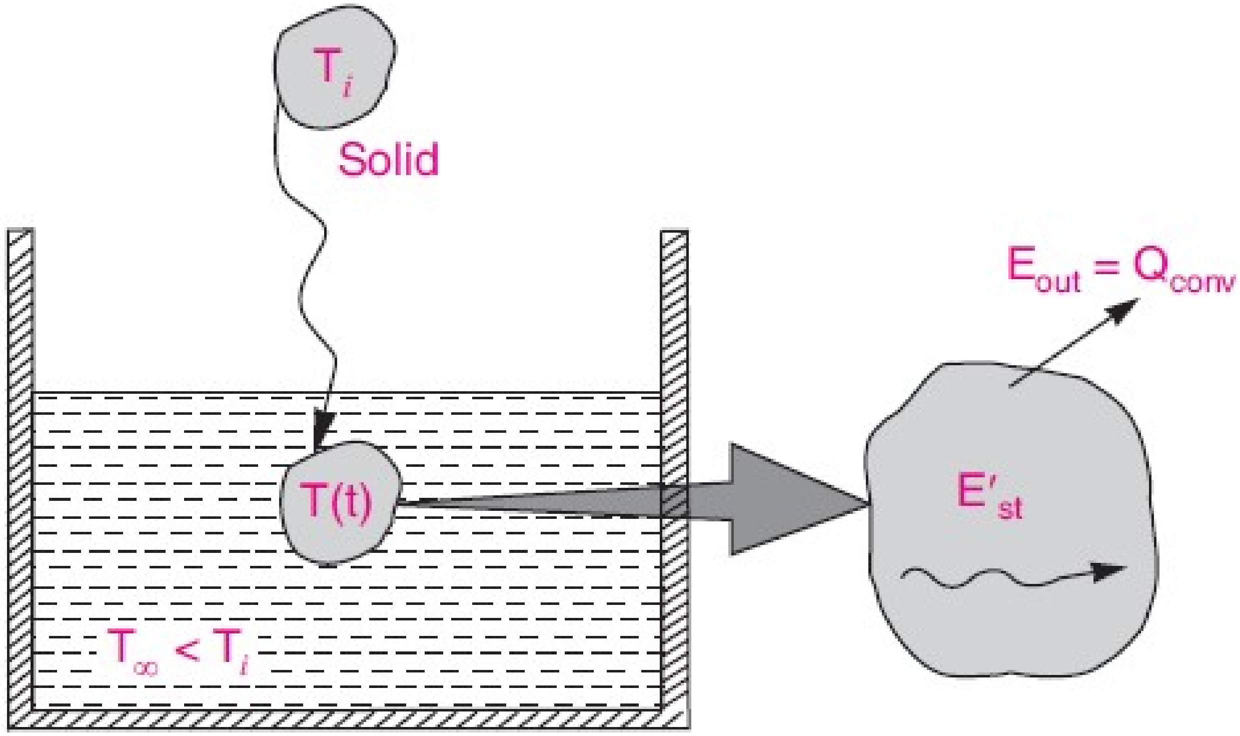


Fig. 6.1. Solid suddenly exposed to convection environment at T_∞

The initial temperature of solid T_i (Fig. 6.1) is greater than ambient fluid temperature, T_∞, the eqn. (6.1) leads to,

$\mathrm{Bi} = \dfrac{h\delta}{k}$, Biot number, a dimensionless number.

$\mathrm{Fo} = \dfrac{\alpha t}{\delta^2}$, Fourier number, a dimensionless number.

$\mathrm{GF} = \dfrac{A_s\delta}{V}$, Geometrical factor, a dimensionless quantity.

The *geometrical factor* GF is considered to be unity for calculation of *characteristic length* δ of the solid as

$$\delta = \frac{V}{A_s} \qquad ...(6.9)$$

Then the temperature distribution eqn. (6.3) within the solid can be expressed as

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left\{-\frac{ht}{\rho\delta C}\right\} \qquad ...(6.10)$$

For certain common body shapes, and their characteristic length δ is shown in Table 6.1

Biot Number

It is defined as *ratio of internal resistance of the solid to heat flow to convection resistance* at the surfaces.

$$\text{Bi} = \frac{\text{Internal resistance to heat flow}}{\text{Convection resistance to heat flow}}$$

$$= \frac{\delta}{kA} \times \frac{hA}{1} = \frac{h\delta}{k} \qquad \text{...(6.16)}$$

It can also be interpreted as the ratio of heat transfer coefficient to the internal specific conductance of the solid. The Biot number is required to determine the validity of the lumped heat capacity approach. The lumped system analysis can only be applied when

$$\text{Bi} \leq 0.1$$

This criteria indicates that the internal resistance of the solid to heat flow is very small in comparison to convection resistance to heat flow at the surfaces.

Fourier Number

It signifies the degree of penetration of heating or cooling effect through the solid. It is defined as the *ratio of the rate of heat conduction to the rate of the thermal energy storage in the solid*. It is denoted by Fo and expressed as

$$\text{Fo} = \frac{kA(\Delta T)/\delta}{\rho VC(\Delta T)/t} = \frac{kAt}{\rho(A\delta)C\delta} = \frac{k}{\rho C}\frac{t}{\delta^2} = \frac{\alpha t}{\delta^2} \qquad \text{...(6.17)}$$

energy which the ...

Example 5.2

A 40 × 40 cm copper slab 5 mm thick at a uniform temperature of 250°C suddenly has its surface temperature lowered at 30°C. Find the time at which the slab temperature becomes 90°C; $\rho = 9000 \text{ kg/m}^3$, $c = 0.38 \text{ kJ/kg K}$, $k = 370 \text{ W/mK}$ and $h = 90 \text{ W/m}^2\text{K}$.

Solution

$$A = 2 \times 0.4 \times 0.4 = 0.32 \text{ m}^2 \text{ (two sides)}$$

$$V = 0.4 \times 0.4 \times 0.005 = 8 \times 10^{-4} \text{ m}^3$$

$$L_c = \frac{V}{A} = L = 2.5 \times 10^{-3} \text{ m}$$

$$B_i = \frac{hL_c}{k} = \frac{(90)\,(2.5 \times 10^{-3})}{370} = 6.1 \times 10^{-4} < 0.1$$

Using Eqn. (5.5),

$$\left(\frac{hA}{\rho c V}\right) = \frac{(90)\,(0.32)}{(9000)\,(380)\,(8 \times 10^{-4})} = 0.0105$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = \exp\left[-\left(\frac{hA}{\rho c V}\right)t\right]$$

$$\frac{90 - 30}{250 - 30} = e^{-0.0105t}$$

$$\frac{60}{220} = e^{-0.0105t}$$

or

$$3.67 = e^{0.0105t}$$

Hence

$$t = 123.83 \text{ s.}$$

Example 5.3

Example 5.3

A stainless steel rod of outer diameter 1 cm originally at a temperature of 320°C is suddenly immersed in a liquid at 120°C for which the convective heat transfer coefficient is 100 W/m^2K. Determine the time required for the rod to reach a temperature of 200°C.

Solution

$$B_i = \frac{hL_c}{k}$$

Taking 1 metre length of wire

$$V = \frac{\pi}{4} D^2 L = \frac{\pi}{4}(0.01)^2 = 7.854 \times 10^{-5}\ \text{m}^3$$

$$A = \pi DL = \pi(0.01) \times 1 = 0.0314\ \text{m}^2$$

$$L_c = \frac{D}{4} = \frac{(0.01)}{4} = 2.5 \times 10^{-3}$$

For stainless steel, take $\rho = 7800\ \text{kg/m}^3$, $c = 460\ \text{J/kg K}$, $k = 40\ \text{W/mK}$

Since $$B_i = \frac{hL_c}{k} = \frac{(100)\ (2.5 \times 10^{-3})}{40} = 6.25 \times 10^{-3} << 0.1,$$

the lumped capacity analysis is applicable. It follows that:

$$\frac{T - T_\infty}{T_0 - T_\infty} = \exp\left[-\left(\frac{hA}{\rho cV}\right) \cdot t\right]$$

Here $$T = 200°\text{C}$$

$$T_0 = 320°\text{C}$$

$$T_\infty = 120°\text{C}$$

$$\frac{hA}{\rho cV} = \frac{4h}{\rho cD} = \frac{4 \times 100}{7800 \times 460 \times 0.01} = 0.01115/\text{s}$$

$\therefore$ $$\frac{200 - 120}{320 - 120} = \frac{80}{200} = e^{-0.01115t}$$

or $$2.5 = e^{+0.01115t}$$

Hence $$t = 82.18\ \text{s}.$$

An aluminium sphere weighing 5.5 kg and initially at a temperature of 290°C is suddenly immersed in a fluid at 15°C. The convective heat transfer coefficient is 58 W/m^2K. Estimate the time required to cool the aluminium to 95°C, using the lumped capacity method of analysis.

Solution

Taking the properties of aluminium as (from Appendix A-1)

$$\rho = 2700 \text{ kg/m}^3$$

$$c = 900 \text{ J/kg K}$$

$$k = 205 \text{ W/mK}$$

$$V = \frac{4}{3}\pi R^3 = \frac{\text{Mass}}{\rho} = \frac{5.5}{2700} = 2.037 \times 10^{-3} \text{ m}^3$$

$\therefore$ $$R = (3V/4\pi)^{1/3} = 0.0786 \text{ m}$$

$$L_c = \frac{R}{3} = 0.0262 \text{ m}$$

Using Eqn. (5.4)

$$\frac{T - T_\infty}{T_0 - T_\infty} = \exp\left[-\left(\frac{hA}{\rho c V}\right)\cdot t\right]$$

We have

$$T = 95°\text{C}$$

$$T_\infty = 15°\text{C}$$

$$T_0 = 290°\text{C}$$

$$\frac{hA}{\rho c V} = \frac{3h}{\rho c R} = \frac{3 \times 58}{2700 \times 900 \times 0.0786} = 9.1 \times 10^{-4}/\text{s}$$

$$\frac{95 - 15}{290 - 15} = \frac{80}{275} = \exp(-9.1 \times 10^{-4} t)$$

or $$3.4375 = \exp(9.1 \times 10 t)$$

Hence $$t = 1357 \text{ s}.$$

Example 6.1. *In a quenching process, a copper plate of 3 mm thick is heated upto 350°C and then suddenly, it is dropped into a water bath at 25°C. Calculate the time required for the plate to reach the temperature of 50°C. The heat transfer coefficient on the surface of the plate is 28 W/m^2.K. The plate dimensions may be taken as length 40 cm and width 30 cm.*

Also calculate the time required for infinite long plate to cool to 50°C. Other parameters remain same.

Take the properties of copper as

$C = 380$ J/kg.K, $\rho = 8800$ kg/m^3,

$k = 385$ W/m.K. (J.N.T.U., May 2004)

Solution

Given : The quenching of a copper plate in water bath.

Size = 40 cm × 30 cm, L = 3 mm,

$T_i = 350°C$, $T_\infty = 25°C$,

$T_i = 350°C$, $T_\infty = 25°C$,

$T = 50°C$, $h = 28 \text{ W/m}^2\text{.K}$,

$C = 380 \text{ J/kg.K}$, $\rho = 8800 \text{ kg/m}^3$,

$k = 385 \text{ W/m.K}$.

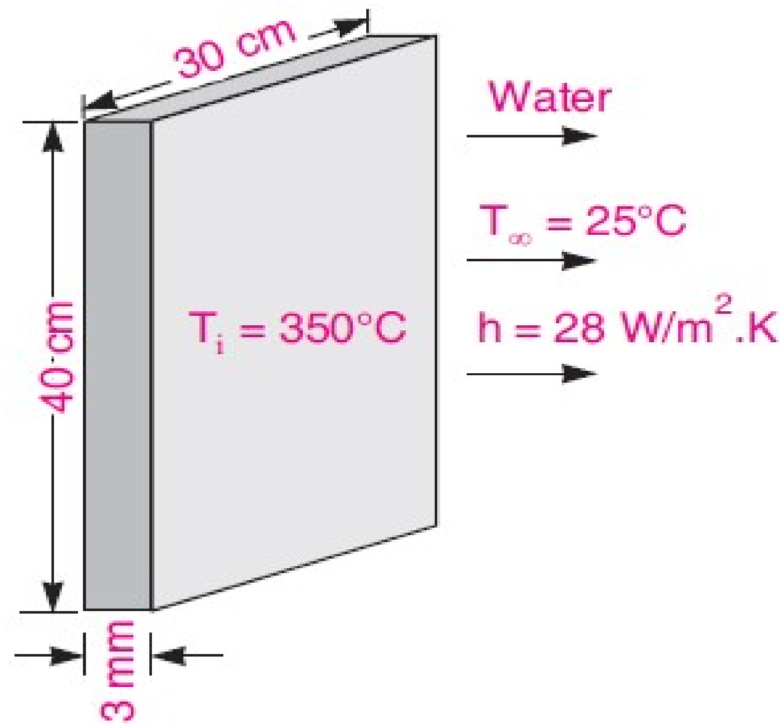


Fig. 6.8. Schematic of plate in example 6.1

To find : Time required to cool the plate to 50°C, if

(*i*) Finite long plate size 40 cm × 30 cm,

(*ii*) Infinite long plate.

Assumptions :

1. The effect of edges of plate for cooling.
2. Internal temperature gradients are negligible.
3. No radiation heat exchange.
4. Constant properties.

Analysis : (*i*) The characteristic length of finite long plate (as shown in Fig. 6.8)

$$\delta = \frac{\text{Volume of plate}}{\text{Exposed area of plate}}$$

$$= \frac{0.4 \times 0.3 \times 0.003}{(2 \times 0.4 + 2 \times 0.3) \times 0.003 + 2 \times 0.3 \times 0.4}$$

$$= 1.474 \times 10^{-3} \text{ m}$$

$$\text{Bi} = \frac{h\delta}{k} = \frac{28 \times 1.474 \times 10^{-3}}{385} = 1.072 \times 10^{-4}$$

which is much smaller than 0.1, thus the lumped system analysis can be applied with reasonable accuracy. Using eqn. (6.10) ;

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left\{-\frac{ht}{\rho C\delta}\right\}$$

Using numerical values.

$$\frac{50 - 25}{350 - 25} = \exp\left\{-\frac{28t}{8800 \times 380 \times 1.474 \times 10^{-3}}\right\}$$

or $$t = -\frac{8800 \times 380 \times 1.474 \times 10^{-3}}{28} \times \ln\left(\frac{25}{325}\right)$$

$$= 451.5 \text{ s} = \mathbf{7.52 \text{ min.}}$$ **Ans.**

(*ii*) Characteristic length of infinite long plate eqn. (6.11)

$$\delta = \frac{L}{2} = 0.0015 \text{ m}$$

$$B_i = \frac{h\delta}{k} = \frac{28 \times 0.0015}{385} = 1.09 \times 10^{-4}$$

which is much less than 0.1, therefore, using lumped system analysis.

$$\frac{50 - 25}{350 - 25} = \exp\left[-\frac{28t}{8800 \times 380 \times 0.0015}\right]$$

or t = 459.5 s = **7.65 min. Ans.**

Example 6.2. *A solid steel ball 5 cm in diameter and initially at 450°C is quenched in a controlled environment at 90°C with convection coefficient of 115 W/m^2.K. Determine the time taken by centre to reach a temperature of 150°C. Take thermophysical properties as*

$$C = 420\ J/kg.K, \quad \rho = 8000\ kg/m^3,$$

$$k = 46\ W/m.K.$$

(P.U., May 2002)

Solution

Given : A solid steel ball quenching with

$T = 150°C$,	$T_\infty = 90°C$,
$T_i = 450°C$,	$h = 115\ W/m^2.K$,
$C = 420\ J/kg.K$,	$\rho = 8000\ kg/m^3$,
$k = 46\ W/m.K$,	$D = 5\ cm = 0.05\ m.$

Fig. 6.9. Schematic for example 6.2

To find : Time required by steel ball to reach 150°C.

To find : Time required by steel ball to reach 150°C.

Assumptions :

1. Internal temperature gradients are negligible.
2. No radiation heat exchange.
3. Constant properties.

Analysis : The characteristic length of the steel ball

$$\delta = \frac{V}{A_s} = \frac{D}{6} = \frac{0.05}{6}\ \text{m} = \left(\frac{0.05}{6}\right)\ \text{m}$$

The Biot number

$$\text{Bi} = \frac{h\delta}{k} = \frac{(115\ \text{W/m}^2.\text{K})}{(46\ \text{W/m.K})} \times \left(\frac{0.05}{6}\ \text{m}\right) = 0.0208$$

which is less than 0.1, hence the lumped heat capacity system analysis may be applied.

Using eqn. (6.10) for temperature distribution

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left\{-\frac{ht}{\rho\delta C}\right\}$$

Substituting the values

$$\frac{150 - 90}{450 - 90} = \exp\left\{-\frac{115 \times 6t}{8000 \times 0.05 \times 420}\right\}$$

or $\quad \ln(60/360) = -(690/168000)t$

or $\quad t = 440.35 \text{ s} = \mathbf{7.34\ min.}$ **Ans.**

Example 6.3. *A titanium alloy blade of an axial compressor for which $k = 25$ W/m.K, $\rho = 4500$ kg/m^3 and $C = 520$ J / kg.K is initially at 60°C. The effective thickness of the blade is 10 mm and it is exposed to gas stream at 600°C, the blade experiences a heat transfer coefficient of 500 W/m^2.K. Use low Biot number approximation to estimate the temperature of blade after 1, 5, 20 and 100 s.*

(N.M.U., May 2002)

Solution

Given : A titanium alloy blade of compressor with

$k = 25$ W/m.K, $\rho = 4500$ kg/m^3,

$C = 520$ J/kg.K, $h = 500$ W/m^2.K,

$T_i = 60°C$, $T_\infty = 600°C$,

$L = 10$ mm, $t = 1, 5, 20$ and 100 s.

To find : Temperature attained by compressor blade after 1, 5, 20 and 100 seconds.

Assumptions: 1. Compressor blade as an infinite wall.

2. Negligible internal temperature gradient

3. No. radiation heat exchange.

4. Constant properties.

Analysis : The characteristic length of blade

$$\delta = \frac{L}{2} = \frac{10}{2} = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$$

The Biot number

$$Bi = \frac{h\delta}{k} = \frac{500 \times 5 \times 10^{-3}}{25} = 0.1$$

Hence it is possible to use the low Biot number approximation

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp\left(-\frac{ht}{\rho\delta C}\right)$$

After 1 s

$$\frac{T - 600}{60 - 600} = \exp\left(-\frac{500 \times 1}{4500 \times 5 \times 10^{-3} \times 520}\right)$$

or $$T = 600 + (-540) \times \exp(-0.0427)$$

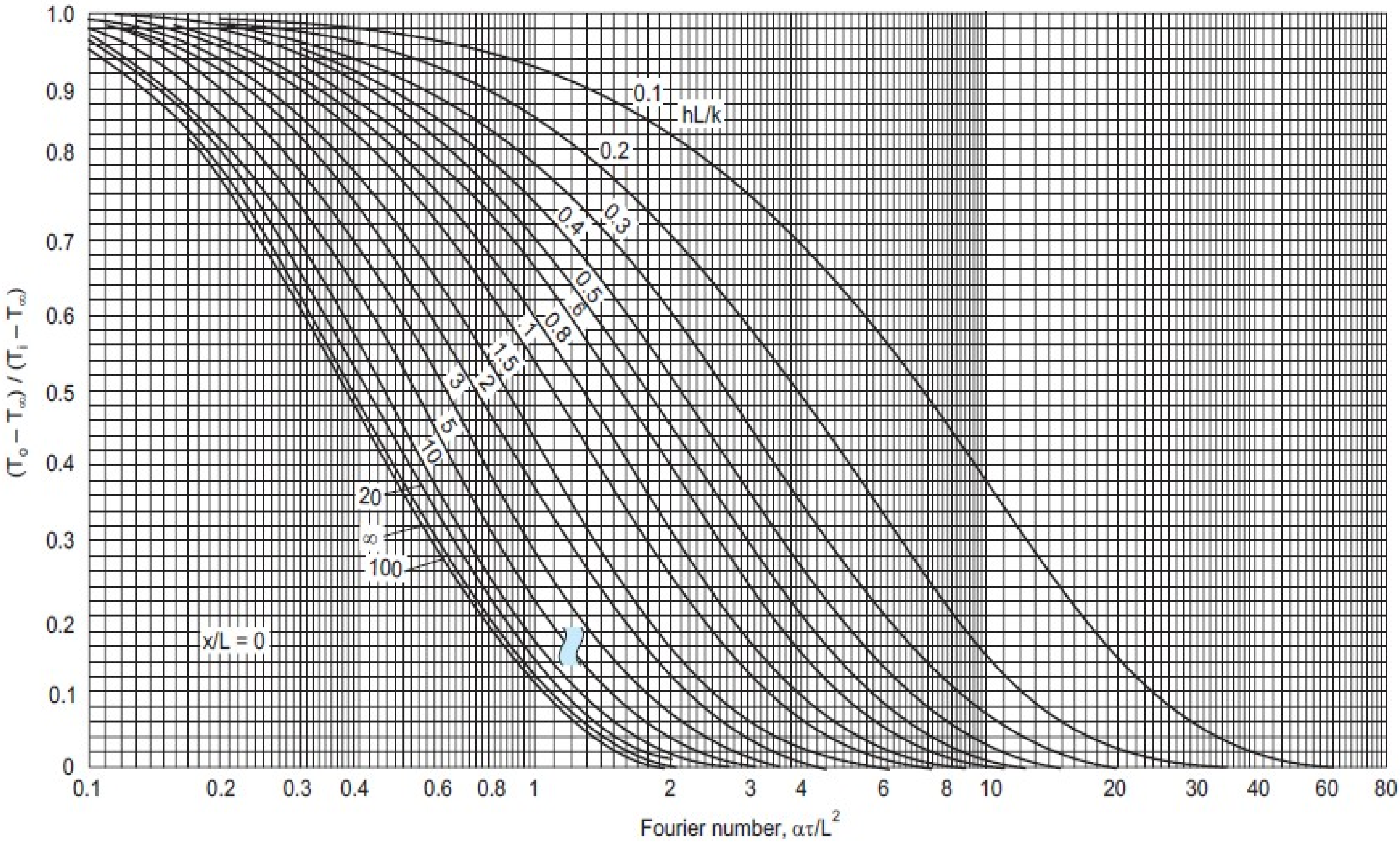
$$= 600 - 540 \times 0.9581 = \mathbf{82.6°C.}$$ **Ans.**

similarly the temperature after

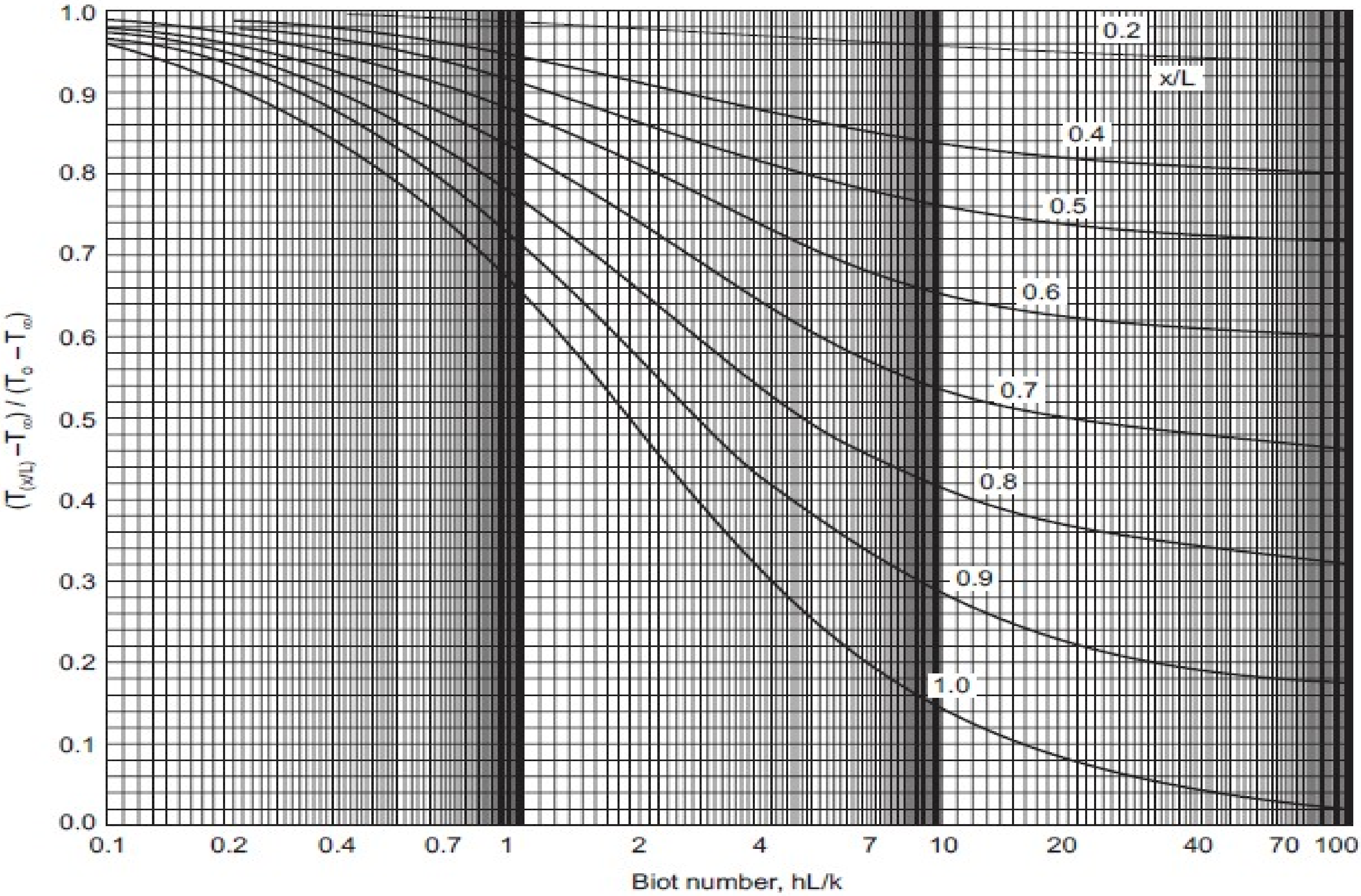
t	T
5 s	163.9°C
20 s	370.3°C
100 s	592.5°C. **Ans.**

Heisler Charts

Centre Temp. Chart—Infinite Plate—Temperature—Time History at Mid Plane

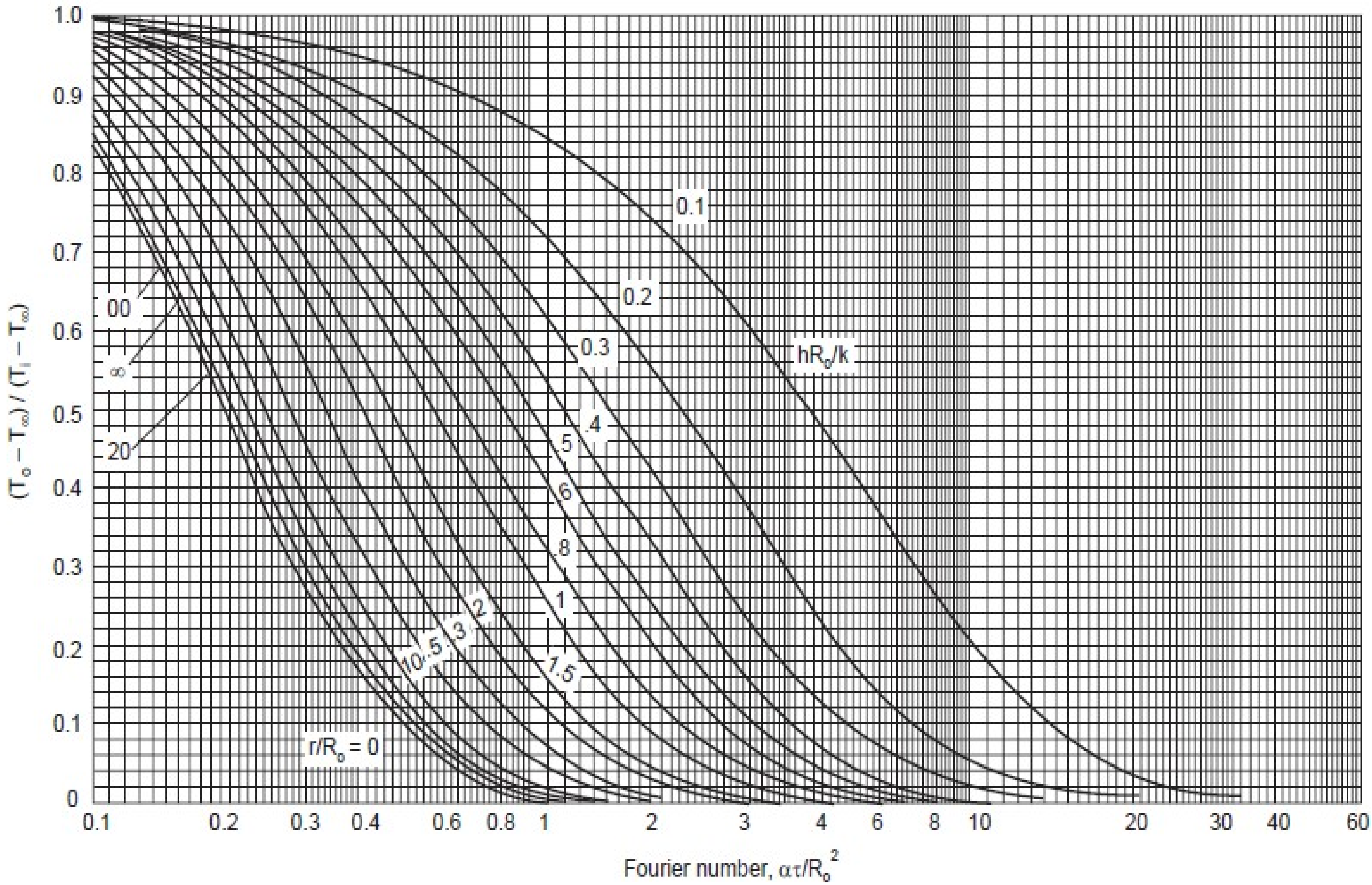


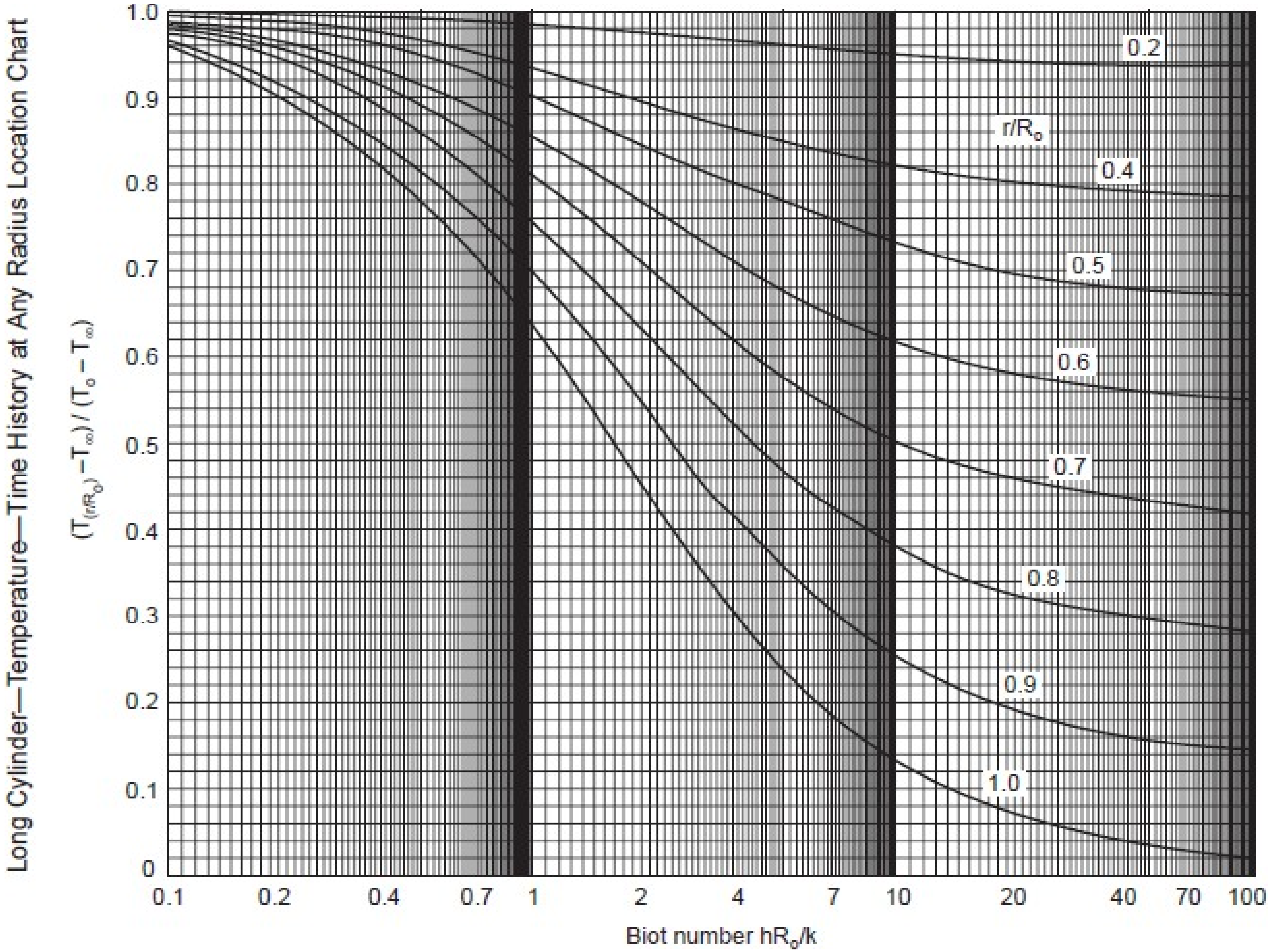
Location Temp. Chart Infintie Plate—Temperature—Time History at any Position



The procedure is as follows: For a given slab and time specification and specification of urroundings Fourier number and Biot numbers are calculated. The centre temperature chart

Long Cylinder—Temperature—Time History at Centreline Centre Temperature Chart





Long Cylinder—Temperature—Time History at Any Radius Location Chart
$(T_{(r/R_o)} - T_\infty) / (T_o - T_\infty)$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.4
0.7
1
2
4
7
10
20
40
70
100
Biot number hR_o/k
r/R_o
0.2
0.4
0.5
0.6
0.7
0.8
0.9
1.0

A slab of aluminium 10 cm thick is originally in a temperature of 500°C. It is suddenly immersed in a liquid at 100°C resulting in a heat transfer coefficient of 1200 W/m^{2}K. Determine the temperature at the centreline and the surface 1 minute after the immersion. Also calculate the total thermal energy removed per unit area of the slab during this period. The properties of aluminium for the given conditions are

$$\alpha = 8.4 \times 10^{-5}\ \text{m}^2/\text{s}\ ; \quad k = 215\ \text{W/mK}\ ;$$

$$\rho = 2700\ \text{kg/m}^3\ ; \quad c = 0.9\ \text{kJ/kg.K.}$$

$T_0 = 500°C$.
$T_\infty = 100°C$

Solution

The Heisler charts of Figs 5.7 to 5.9 may be used for solving this problem.

Here $\quad 2L = 10\ \text{cm}, \quad L = 5\ \text{cm}, \quad t = 1\ \text{min} = 60\ \text{s}$

$$\frac{\alpha t}{L^2} = \frac{(8.4 \times 10^{-5})\,(60)}{(0.05)^2} = 2.016$$

$$\frac{1}{B_i} = \frac{k}{hL} = \frac{215}{(1200)\,(0.05)} = 3.583$$

From Fig. 5.7 the centre line temperature is given by

$$\frac{T_{(0,t)} - T_\infty}{T_0 - T_\infty} = \frac{\theta_c}{\theta_0} = 0.68$$

$\therefore$
$$\theta_c = T_{(0,t)} - T_\infty = 0.68(500 - 100) = 272$$

or
$$T_{(0,t)} = 272 + 100 = 372°\text{C}$$

For the temperature at the surface

$$\frac{x}{L} = 1.0$$

From Fig. 5.8 at $x/L = 1.0$ and for $k/hL = 3.583$

$$\frac{T_{(x,t)} - T_\infty}{T_{(0,t)} - T_\infty} = 0.880$$

r

$$T_{(x,t)} = (0.88)(372 - 100) + 100 = 339.36°\text{C}$$

To calculate the energy loss

$$\frac{h^2 \alpha t}{k^2} = \frac{(1200)^2 (8.4 \times 10^{-5})(60)}{(215)^2} = 0.157$$

$$B_i = \frac{hL}{k} = \frac{(1200)(0.05)}{215} = 0.28$$

From Fig. 5.9,

$$\frac{U}{U_0} = 0.32$$

For unit area

$$\frac{U_0}{A} = \frac{\rho c V (T_0 - T_\infty)}{A} = \rho c(2L)(T_0 - T_\infty)$$

$$= (2700)(900)(0.1)(400) = 97.2 \times 10^6 \text{ J/m}^2$$

∴ Heat removed per unit surface area is

$$\frac{U}{A} = 0.32 \times 97.2 \times 10^6 = 31.1 \times 10^6 \text{ J/m}^2.$$

Example 6.9: *A slab of thickness 15 cm initially at 30°C is exposed on one side to gases at 600°C with a convective heat transfer coefficient of 65W/m²K. The other side is insulated. Using the following property values determine the temperatures at both surfaces and the centre plane after 20 minutes, density: 3550 kg/m³, sp. heat = 586 J/kgK, conductivity = 19.5 W/mK. Also calculate the heat flow upto the time into the solid.*

Solution: The data is presented in Fig. 6.13(*a*). The slab model with the centre plane at zero and thickness 0.15 m is used. As inside is insulated this can be considered as half slab with $x = 0$ at insulated face.

The quantities Bi and Fo are calculated using

$$Bi = \frac{65 \times 0.15}{19.5} = 0.5,$$

$$Fo = \frac{19.5}{3550 \times 586} \times 20 \times 60/0.15 \times 0.15 = 0.5$$

The procedure of obtaining temperature is illustrated with skeleton charts in Fig. 6.13 (*b*) and (*c*). The centre temperature is obtained by entering the chart as shown in Fig. 6.13 (*b*). The excess temperature ratio at the centre is obtained as 0.864.

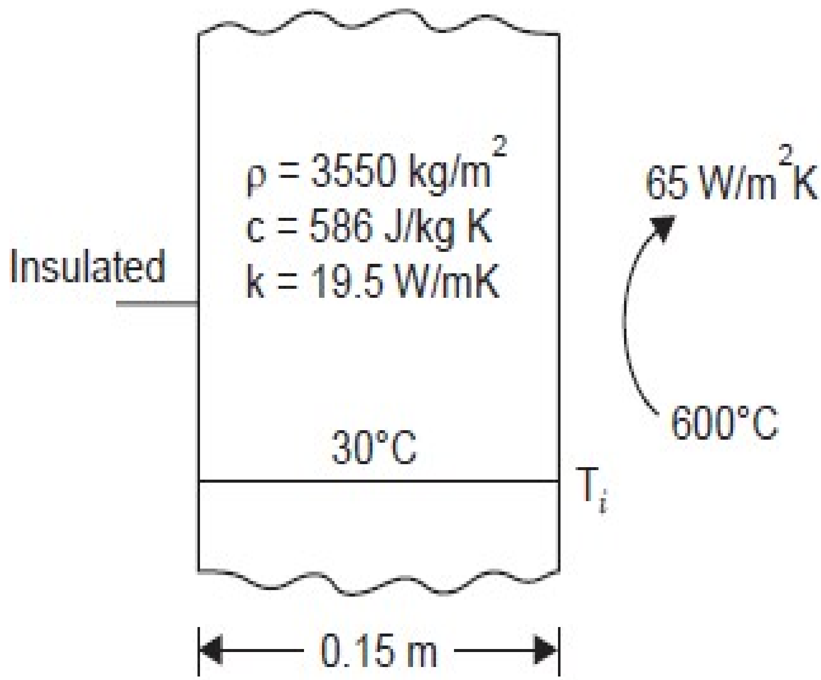


Fig. 6.13 (a) Model.

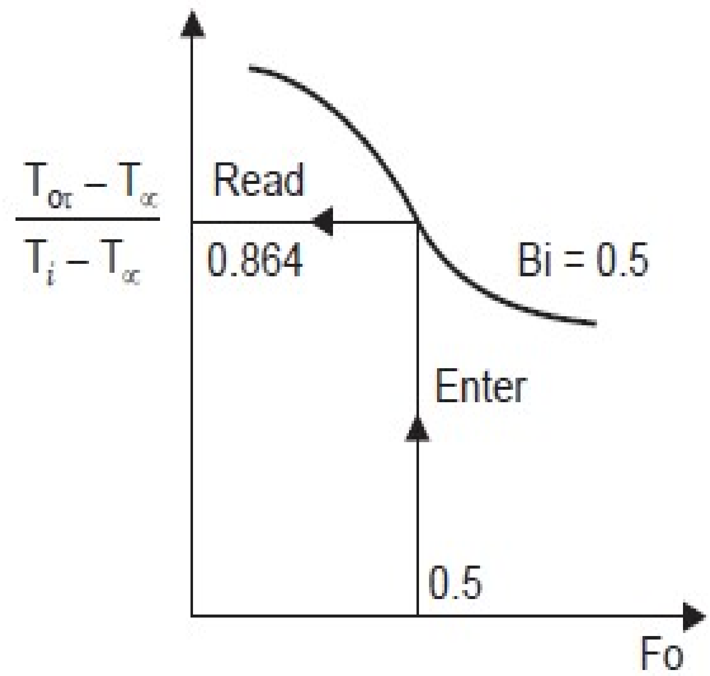


Fig. 6.13 (b)

$$\frac{T_{o,\tau} - T_\infty}{T_i - T_\infty} = 0.864, \text{ after } 20 \text{ minutes}$$

$$\frac{T_{o,\tau} - 600}{30 - 600} = 0.864 \quad \therefore \quad \mathbf{T_{o,\tau} = 107.52°C}$$

To obtain the surface and mid plane temperatures, the location chart is entered at $Bi = 0.5$ as schematically shown in Fig. 6.13 (*c*) and the values at $x/L = 1$ and 0.5 are read as 0.792 and 0.948.

The surface temperature is given by

$$\frac{T_{L,\tau} - T_\infty}{T_i - T_\infty} = 0.792 \times 0.864$$

$$\frac{T_{L,\tau} - 600}{30 - 600} = 0.6843$$

∴ **Surface temperature $T_L = 210°C$**

The mid plane temperature:

$$\frac{T_{x,\tau} - 600}{30 - 600} = 0.864 \times 0.948$$

∴ **$T = 133.13°C$**

The heat flow is determined using the heat flow chart as shown schematically in Fig. 6.13(d). First the parameter is calculated:

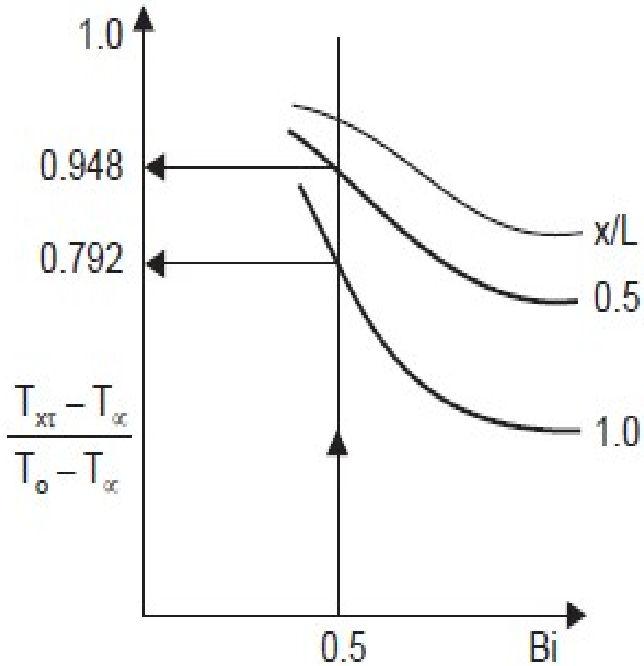


Fig. 6.13 (c)

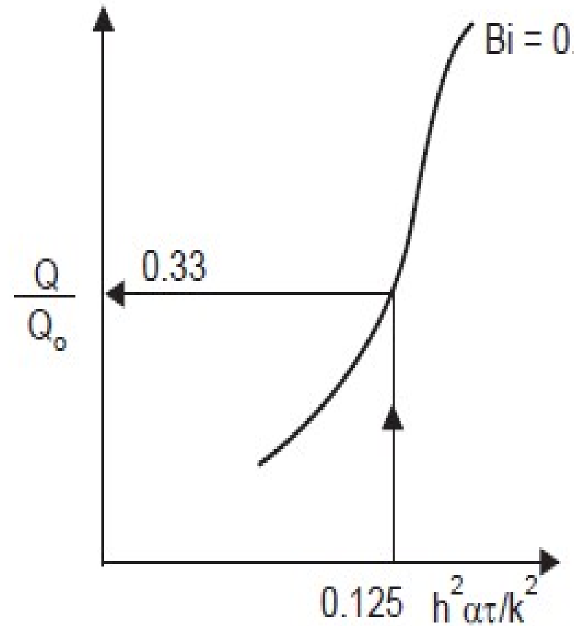


Fig. 6.13 (d)

$$\frac{h^2\alpha\tau}{k^2} = \frac{65 \times 65 \times 19.5 \times 20 \times 60}{3550 \times 568 \times 19.5^2} = 0.125$$

Entering the chart at this point and finding the meeting of point with $Bi = 0.5$, the ratio Q/Q_o is read as 0.33.

$$\therefore \qquad \mathbf{Q} = 0.33 \times 3350 \times 586 \times 0.15 \times 1(600 - 30)$$

$$= \mathbf{55.39 \times 10^6\ J/m^2}$$

A rough check can be made by using an average temperature increase and finding the change in internal energy. The average temperature rise is $(107.52 + 210 + 133.13)/3 - 30 = 120.22°C$.

$$\mathbf{Q} = 3350 \times 0.15 \times 586 \times 120.22 = \mathbf{37.51 \times 10^6}$$

This is of the same order of magnitude and hence checks.

II - UNIT

CONVECTION

The process of heat transfer between a surface and a fluid flowing in contact with it is called **convection.** If the flow is caused by an **external device** like a pump or blower, it is termed as **forced convection.** If the flow is caused by the **buoyant forces** generated by heating or cooling of the fluid the process is called as **natural or free convection.**

In the previous chapters the heat flux by convection was determined using equation.

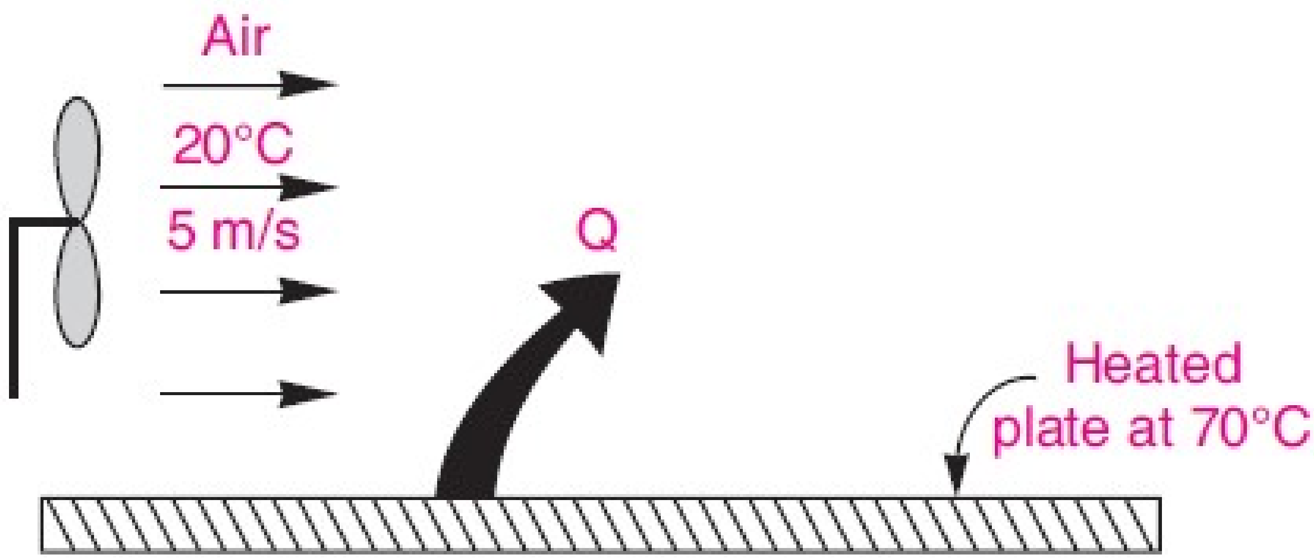
$$q = h\,(T_s - T_\infty) \qquad ...(7.1)$$

q is the heat flux in W/m^2, T_s is the surface temperature and T_∞ is the fluid temperature of the free stream, the unit being °C or K. Hence the unit of convective heat transfer coefficient h is $W/m^2\,K$ or $W/m^2\,°C$ both being identically the same.

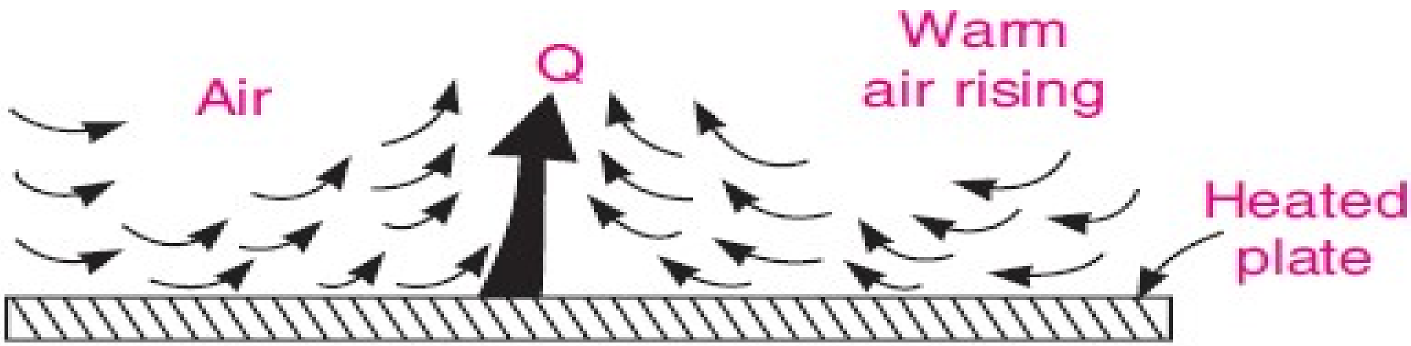
CLASSIFICATION OF CONVECTION

The convection heat transfer is classified as *natural* (or *free*) or *forced convection*, depending on how the fluid motion is initiated. The natural or *free convection* is a process, in which the fluid motion results from heat transfer. When a fluid is heated or cooled, its density changes and the buoyancy effects produce a natural circulation in the affected region, which causes itself the rise of warmer fluid and the fall of colder fluid : Therefore, energy transfers from hotter region to colder region and such process is repeated as long as the temperature difference in the fluid exists.

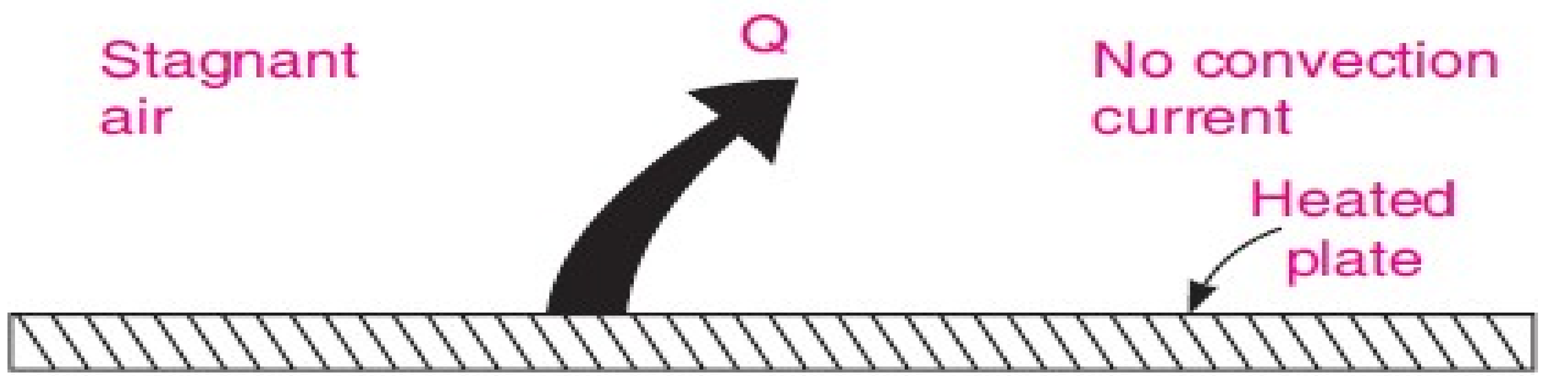
In the *forced convection*, the fluid is forced to flow over a surface or in a duct by external means such as a pump or a fan. A large number of heat transfer applications utilize forced convection, because the heat transfer rate is much faster than that in free convection.



(a) Forced convection



(b) Natural convection



(c) In absence of fluid motion, heat transfer in the fluid is by conduction only

Fig. 7.1. The heat transfer from a hot surface to the surrounding fluid

Velocity Boundary Layer

Consider the flow of fluid over a flat plate as shown in Fig. 7.5. The fluid approaches the plate in x direction with a uniform velocity u_∞. The fluid particles in the fluid layer adjacent to the surface get zero velocity. This motionless layer acts to retard the motion of particles in the adjoining fluid layer as a result of friction between the particles of these two adjoining fluid layers at two different velocities. This fluid layer then acts to retard the motion of particles of next fluid layer and so on, until a distance $y = \delta$ from the surface reaches, where these effects become negligible and the fluid velocity u reaches the free stream velocity u_∞. As a result of frictional effects between the fluid layers, the local fluid velocity u will vary from $x = 0$, $y = 0$ to $y = \delta$.

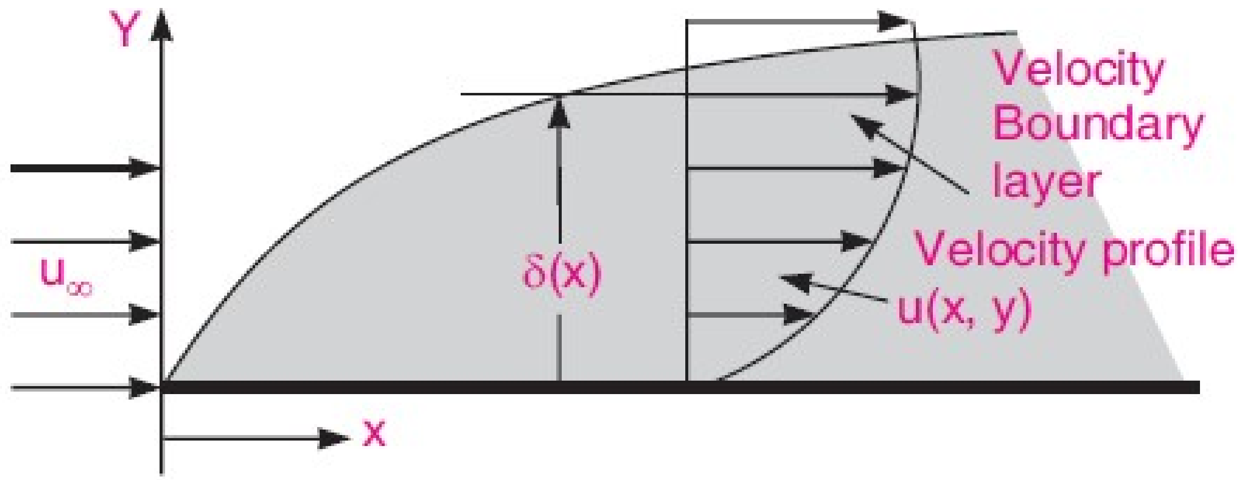


Fig. 7.5. Velocity boundary layer on a flat plate

The region of the flow over the surface bounded by δ in which the effects of viscous shearing forces caused by fluid viscosity are observed, is called the **velocity boundary layer or hydrodynamic boundary layer** *or simply the* **boundary layer.** The thickness of boundary layer δ is generally defined as a distance from the surface at which local velocity $u = 0.99$ of free stream velocity u_∞.

7.5.1. Laminar Boundary Layer

The velocity boundary layer starts at the leading edge of the plate as a laminar boundary layer, in which the fluid motion is highly ordered and it is possible to identify the stream lines along which particles move. The fluid motion along a stream line is characterized by the velocity components u and v in both x and y directions and it influences the momentum and energy transfer through the boundary layer. The velocity profile in laminar boundary layer is approximately parabolic.

7.5.2. Turbulent Boundary Layer

The fluid motion in the turbulent boundary layer has very large disturbances and is characterized by velocity fluctuations. The fluctuations increase the momentum and heat transfer. Due to fluid mixing, the turbulent boundary layer thickness is larger and velocity profiles are flatter with the sharp drop near the surface.

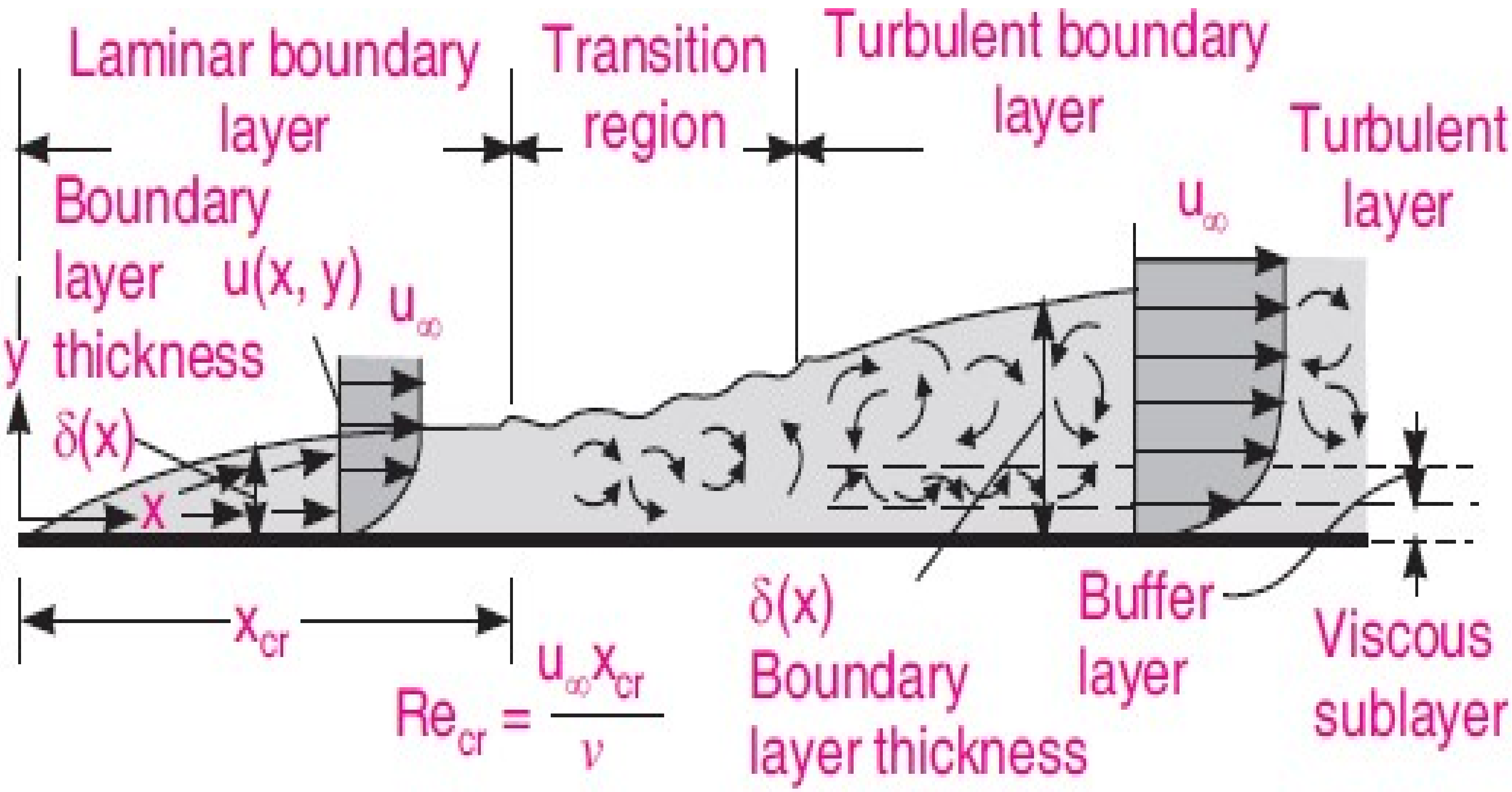


Fig. 7.8. Boundary layer concept for flow along a flat plate

Air at 20°C is flowing along a heated flat plate at 134°C at a velocity of 3 m/s. The plate is 2 m long and 1.5 m wide. Calculate the thickness of the hydrodynamic boundary layer and the skin friction coefficient at 40 cm from the leading edge of the plate. The kinematic viscosity of air at 20°C may be taken at 15.06×10^{-6} m²/s.

Solution

At $$x = 40 \text{ cm}; \; Re_x = \frac{u_\infty x}{\nu} = \frac{(3)\,(0.4)}{15.06 \times 10^{-6}} = 7.9 \times 10^4 < 5 \times 10^5$$

So the boundary layer is laminar. Its thickness is calculated from Eqn. (7.13),

$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{(5)(0.4)}{(7.9 \times 10^4)^{1/2}} = 0.71 \times 10^{-2} m = 7.1\text{mm}$$

The local skin friction coefficient is given by Eqn. (7.14)

$$C_{fx} = \frac{0.664}{\sqrt{Re_x}} = \frac{0.664}{(7.9 \times 10^4)^{1/2}} = 2.36 \times 10^{-3}.$$

Example 7.2

For the flow system in Example 7.1 calculate the local that transfer coefficient at $x = 0.4$ m and the heat transferred from the first 40 cm of the plate.

Solution

The film temperature, $T_f = \dfrac{134 + 20}{2} = 77°C$

The physical properties of air at 77°C are

$$\rho = 0.998 \text{ kg/m}^3,\ C_p = 1.009 \text{ kJ/kg°C},\ \nu = 20.76 \times 10^{-6} \text{ m}^2\text{/s}$$

$$k = 0.03 \text{ W/mK},\ Pr = 0.697$$

$$x = 0.4 \text{ m}$$

$$Re_x = \frac{u_\infty \cdot x}{\nu} = \frac{(3)\,(0.4)}{20.76 \times 10^{-6}} = 5.78 \times 10^4$$

$$Nu_x = \frac{h_x x}{k} = 0.332\, Re_x^{1/2}\, Pr^{1/3}$$

$$= (0.332)\,(5.78 \times 10^4)^{1/2}\,(0.697)^{1/3} = 70.6$$

$$h_x = \frac{(70.6)\,(0.03)}{0.4} = 5.3\ \text{W/m}^2\text{K}$$

$$\frac{K.Nu.L}{L}$$

The average value of the heat transfer coefficient is twice this value or

$$\bar{h}_x = (2)\,(5.3) = 10.6\ \text{W/m}^2\text{K}$$

The heat flow is

$$Q = \bar{h}_x A (T_s - T_\infty)$$

$$= (10.6)\,(0.4)\,(1.5)\,(134 - 20) = 725\ \text{W}$$

The heat flow from the both sides of the plate = (2) (725) = 1450 W.

Air at a pressure of 8 kN/m^2 and a temperature at 250°C flows over a flat plate 0.3 m wide and 1 m long at a velocity of 8 m/s. If the plate is to be maintained at a temperature of 78°C estimate the rate of heat to be removed continuously from the plate.

Solution

The film temperature, $T_f = \frac{1}{2}(T_s + T_\infty)$

$$= \frac{250 + 78}{2} = 164°C = 437 \text{ K}$$

The physical properties of air at (437 K, p = 1 atm) are

$$\nu = 30.8 \times 10^{-6} \text{ m}^2/\text{s}, \quad k = 36.4 \times 10^{-3} \text{ W/mK},$$

$$Pr = 0.69$$

$$C_p = 1.018 \text{ kJ/kg°C}$$

The properties of air such as k, μ and P_r do not change much with pressure but the density of air does change a lot. Using the perfect gas equation

$$\rho = p/RT,$$

the kinematic viscosity, $\nu = \frac{\mu}{\rho}$ will vary with pressures as $\frac{\nu_1}{\nu_2} = \frac{p_2}{p_1}$ (at constant temperature).

Hence the kinematic viscosity of air at 437 K and $p = 8$ kN/m^2 would be

$$= 30.8 \times 10^{-6} \times \frac{1.0133 \times 10^5}{8 \times 10^3} = 3.90 \times 10^{-4}\ \text{m}^2/\text{s}$$

for the given plate

$$Re_L = \frac{u_\infty L}{\nu} = \frac{(8)\ (1)}{3.9 \times 10^{-4}} = 2.05 \times 10^4$$

Hence the flow is laminar over the entire length of the plate.

Using Eqn. (7.40)

$$\bar{h} = 2h_x = 0.662\left(\frac{k}{L}\right) Re_L^{1/2}\ Pr^{1/3}$$

$$= \frac{(0.662)(36.4 \times 10^{-3})(2.05 \times 10^{4})^{1/2} \times (0.69)^{1/3}}{(1)}$$

$$= 3.04 \text{ W/m}^2\text{K}$$

Since the plate has two surfaces from which heat is to be removed, the rate of heat removal is

$$Q = 2hA(T_\infty - T_s)$$

$$= (2)(3.04)(1)(0.3)(250 - 78) = 313.7 \text{ W}.$$

Example ...

A flat plate 1.0 m wide and 1.0 m long is placed in a wind tunnel. The temperature and velocity of free stream air are 10°C and 80 m/s respectively. The flow over the whole length of the plate is made turbulent with the help of a turbulizing grid placed upstream of the plate. Determine the thickness of the boundary layer at the trailing edge of the plate. Also calculate the mean value of the heat transfer coefficient from the surface of the plate.

Solution

The physical properties of air at 10°C are

$$k = 0.025 \text{ W/mK}, \quad \nu = 14.15\times10^{-6}\,\text{m}^2/\text{s}, \quad Pr = 0.72$$

$$Re_L = \frac{u_\infty \cdot L}{\nu} = \frac{(80)\,(1)}{14.15\times10^{-6}} = 5.65\times10^{6} > 5\times10^{5}$$

Due to turbulizing grid, the flow on the plate becomes turbulent right from its leading edge and remains so over the entire plate. The turbulent boundary layer at the trailing edge $x = L$ can be calculated from Eqn. (7.54)

$$\delta = 0.381\, L Re_L^{-1/5} = \frac{(0.381)\,(1)}{(5.65 \times 10^6)^{1/5}} = 0.0170 \text{ m} = 1.70 \text{ cm}$$

The mean value of the Nusselt number is given by Eqn. (7.60)

$$\overline{Nu}_L = 0.037\, Re_L^{4/5}\, Pr^{1/3}$$

$$= (0.037)\,(5.65 \times 10^6)^{4/5}\,(0.72)^{1/3} = 8363$$

$$\therefore \qquad \bar{h} = \overline{Nu}_L \cdot \frac{k}{L} = \frac{(8363)\,(0.025)}{1} = 209 \text{ W/m}^2\text{K}.$$

Example 7.7

An air stream at 0°C is flowing along a heated plate at 90°C at a speed of 75 m/s. The plate is 45 cm long and 60 cm wide. Assuming the transition of boundary layer to take place at $Re_{x,c} = 5 \times 10^5$ calculate the average values of friction coefficient and heat transfer coefficient for the full length of the plate. Hence calculate the rate of energy dissipation from the plate.

Solution

Film temperature, $T_f = \dfrac{90 + 0}{2} = 45°\text{C}.$

The properties of air at 45°C are

$$k = 2.8 \times 10^{-2} \text{ W/mK}, \quad \nu = 17.45 \times 10^{-6} \text{m}^2/\text{s}, \quad Pr = 0.698$$

Now $$Re_{x,c} = \frac{u_\infty x_c}{\nu} = 5 \times 10^5$$

∴ $$x_c = \frac{(5 \times 10^5)\,(17.45 \times 10^{-6})}{(75)} = 0.116 \text{ m} = 11.6 \text{ cm}$$

So laminar flow exists up to a length of 11.6 cm and the turbulent flow thereafter

$$Re_L = \frac{(75)\,(0.45)}{(17.45 \times 10^{-6})} = 1.93 \times 10^6$$

The average value of the friction coefficient is given by Eqn. (7.59) as

$$\overline{C_{fL}} = \frac{0.074}{(Re_L^{1/5})} - \frac{1740}{Re_L}$$

$$= \frac{0.074}{(1.93 \times 10^6)^{1/5}} - \frac{1740}{1.93 \times 10^6} = 4.09 \times 10^{-3} - 0.9 \times 10^{-3}$$

$$= 3.19 \times 10^{-3}$$

The average heat transfer coefficient can be calculated from Eqn. (7.58) as

$$\overline{Nu}_L = (0.037\, Re_L^{4/5} - 870)\, Pr^{1/3}$$

$$= \left[(0.037)\,(1.93 \times 10^6)^{4/5} - 870\right](0.698)^{1/3}$$

$$= 2732$$

$$\therefore \quad \bar{h}_L = \frac{(2732)\,(2.8 \times 10^{-2})}{0.45} = 170 \text{ W/m}^2\text{K}$$

The rate of energy dissipation from the plate.

$$Q = 2\bar{h}_L A (T_s - T_\infty)$$

$$= (2)\,(170)\,(0.45)\,(0.6)\,(90) = 8262 \text{ W} = 8.262 \text{ kW}$$

Assuming that a man can be represented by a cylinder 30 cm in diameter and 1.7 m high with a surface temperature of 30°C, calculate the heat he would lose while standing in a 36 km/h wind at 10°C.

Solution

The film temperature, $T_f = \dfrac{30+10}{2} = 20°C$

P, 33

The physical properties of air at 20°C are:

$$k = 2.59 \times 10^{-2}\ \text{W/mK},\quad \nu = 15.00 \times 10^{-6}\ \text{m}^2/\text{s};\quad Pr = 0.707$$

$$\text{The speed of wind} = \frac{36 \times 1000}{3600} = 10\ \text{m/s}$$

$$Re_D = \frac{uD}{\nu} = \frac{(10)\,(30 \times 10^{-2})}{15.00 \times 10^{-6}} = 2 \times 10^5$$

Employing Eqn. (7.65)

$$\overline{Nu}_D = \frac{\bar{h}D}{k} = C(Re_D)^n\,(Pr^{1/3})$$

where $C = 0.027$ and $n = 0.805$ (from Table 7.2).

The Nusselt number is then

$$\overline{Nu}_D = (0.027)\,(2 \times 10^5)^{0.805}\,(0.707)^{0.333} = 444.7$$

$$\bar{h} = \overline{Nu}_D \cdot \frac{k}{D} = \frac{(444.7)\,(2.59 \times 10^{-2})}{(30 \times 10^{-2})} = 38.39 \text{ W/m}^2\text{K}$$

The rate of heat lost by the man $= \bar{h}A(T_s - T_\infty)$

$$= (38.39)\,(\pi \times 30 \times 10^{-2} \times 1.7)\,(30 - 10) = 1230.2 \text{ W}$$

Example 7.9

Air stream at 27°C is moving at 0.3 m/s across a 100 W electric bulb at 127°C. If the bulb is approximated by a 60 mm diameter sphere, estimate the heat transfer rate and the percentage of power lost due to convection.

Solution

The film temperature, $T_f = \dfrac{127 + 27}{2} = 77°\text{C}$

The physical properties of air at 77°C are

$$\nu = 2.08 \times 10^{-5}\ \text{m}^2/\text{s},\quad k = 0.03\ \text{W/mK},\quad Pr = 0.697$$

$$Re_D = \frac{uD}{\nu} = \frac{(0.3)(60 \times 10^{-3})}{2.08 \times 10^{-5}} = 865.3$$

... gives the Nusselt number as

Equation (7.69) gives the Nusselt number as

$$\overline{Nu}_D = 0.37\, Re_D^{0.6} = \frac{\bar{h}D}{k}$$

$$\therefore \qquad \bar{h} = \frac{k}{D}(0.37)\,(Re_D)^{0.6}$$

$$= \frac{(0.03)\,(0.37)\,(865.3)^{0.6}}{0.06} = 10.7 \text{ W/m}^2\text{K}$$

The heat transfer rate is given by

$$Q = \bar{h}A(T_s - T_\infty) = (10.7)(\pi)\,(0.06)^2\,(127 - 27) = 12.10 \text{ W}$$

The percentage of heat lost by forced convection is therefore

$$= \frac{12.10}{100} \times 100 = 12.10\%.$$

Dimensional Analysis

- Dimensional analysis is used to interpolate the experimental laboratory results (prototype models) to full scale system.
- Two criteria must be fulfilled to perform such an objective:
  - Dimensional similarity, in which all dimensions of the prototype to full scale system must be in the same ratio.
  - Dynamic similarity, in which relevant dimensionless groups are the same between prototype model and full scale system.
- The convective heat transfer coefficient is a function of the thermal properties of the fluid, the geometric configuration, flow velocities, and driving forces.

- Dimensional analysis is a mathematical method which makes use of the study of the dimensions for solving several engineering problems.
- This method can be applied to all types of fluid resistances, heat flow problems and many other problems in fluid mechanics and thermodynamics.
- In dimensional analysis, the various physical quantities used in fluid phenomenon can be expressed in terms of fundamental quantities. These fundamental quantities are mass (M), length (L), time (T), and temperature (θ or t)

For example

$$\text{Force } (F) = (\text{Mass})\ (\text{Acceleration})$$

$$F = (M)\left(L/t^2\right) = MLt^{-2}$$

Similarly

$$\text{Viscosity} = (\text{Shear stress})/(du/dy)$$

$$= (\text{Force/area})/(\text{Velocity/length})$$

$$= \frac{\left(MLt^{-2}\right)/\left(L^2\right)}{(L/t)/(L)}$$

Table 6.1 Some Physical Quantities and their Dimensions

Quantity	*Symbol*	*Dimensions*
Mass	m	M
Length	L, x	L
Time	t	t
Velocity	U	Lt^{-1}
Acceleration	a	Lt^{-2}
Force	F	MLt^{-2}
Work	W	ML^2t^{-2}
Energy heat	E, Q	ML^2t^{-2}
Power	P	ML^2t^{-3}
Density	ρ	ML^{-3}
Pressure, Stress	p, σ	$ML^{-1}t^{-2}$
Viscosity	μ	$ML^{-1}t^{-1}$
Kinematic viscosity	ν	L^2t^{-1}
Specific heat	c	$L^2t^{-2}T^{-1}$
Thermal conductivity	k	$MLt^{-3}T^{-1}$
Thermal diffusivity	α	L^2t^{-1}
Heat Transfer coefficient	h	$Mt^{-3}T^{-1}$
Coefficient of thermal Expansion	β	T^{-1}

Buckingham's π Theorem: This theorem is used as a rule of thumb for determining the number of independent dimensionless groups that can be obtained from a set of variables. By independent dimensionless groups we mean those groups out of a set which cannot be derived by combining the rest of the groups in any manner, whatsoever.

Buckingham's π theorem states that the number of independent dimensionless groups that can be formed from a set of n variables having r basic dimensions is $(n - r)$.

For example, let A_1, A_2, A_3, A_4, A_5 and A_6 be the relevant variables in a problem and let these six variables be expressed in terms of four basic dimensions, (M, L, T, t). The number of independent dimensionless groups representing this phenomenon would then, according to Buckingham's theorem, be $6 - 4 = 2$. The relationship between these dimensionless groups can be expressed as

$$F(\pi_1, \pi_2) = 0 \tag{6.96}$$

The procedure for obtaining the dimensionless groups for forced and free convection will be outlined in the following subsections.

6.9.1 Dimensional Analysis Applied to Forced Convection

Let us now consider the case of a fluid flowing across a heated tube. The various variables pertinent to this problem along with their symbols and dimensions are given in Table 6.2.

Table 6.2 Pertinent Variables in Forced Convection Heat Transfer

Variable	*Symbol*	*Dimension*
Tube diameter (Characteristic length)	D	L
Fluid density	ρ	ML^{-3}
Fluid velocity	U	Lt^{-1}
Fluid viscosity	μ	$ML^{-1}t^{-1}$
Specific heat	C_p	$L^2t^{-2}T^{-1}$
Thermal conductivity	k	$MLt^{-3}T^{-1}$
Heat transfer coefficient	h	$Mt^{-3}T^{-1}$

There are seven variables and four basic dimensions, so three independent dimensionless parameters would be required to correlate the experimental data.

The three dimensionless groups will be symbolised by π_1, π_2 and π_3 and may be obtained by a se procedure. Each dimensionless parameter will be formed by combining a core group of r variables with one of the remaining variables not in the core. The core will include any four (in this case) of the variables which among them, include all of the basic dimensions. We may, arbitrarily choose D, ρ, μ and k as the core. The groups to be formed are now represented as the following π groups

$$\pi_1 = D^a \rho^b \mu^c k^d U$$

$$\pi_2 = D^e \rho^f \mu^g k^i C_p$$

$$\pi_3 = D^j \rho^l \mu^m k^n h$$

Since these groups are to be dimensionless, so the variables are raised to certain exponents $a, b, c, \ldots, m, n$. Starting with π_1, we write dimensionally as

$$M^0 L^0 T^0 t^0 = 1 = (L)^a \left(\frac{M}{L^3}\right)^b \left(\frac{M}{Lt}\right)^c \left(\frac{ML}{t^3 T}\right)^d \left(\frac{L}{t}\right)$$

Equating the sum of the exponents of each basic dimension to zero, we get the following set of equations

For

$$M; \quad 0 = b + c + d$$

$$L; \quad 0 = a - 3b + d + 1 - c$$

$$t; \quad 0 = -c - 3d - 1$$

$$T; \quad 0 = -d$$

Solving these equations, we get

$$d = 0$$
$$c = -1$$
$$b = 1$$
$$a = 1$$

giving $$\pi_1 = \frac{\rho U D}{\mu} = Re_D \text{ (Reynolds number)}$$

Similarly for π_2

$$1 = (L)^e \left(\frac{M}{L^3}\right)^f \left(\frac{M}{Lt}\right)^g \left(\frac{ML}{t^3 T}\right)^i \left(\frac{L^2}{t^2 T}\right)$$

for

$$M;\quad 0 = f + g + 1$$
$$L;\quad 0 = e - 3f - g + i + 2$$
$$t;\quad 0 = -g - 3i - 2$$
$$T:\quad 0 = -i - 1$$

from these we find that $i = -1,\ g = 1,\ f = 0,\ e = 0$; giving

$$\pi_2 = \frac{\mu C_p}{k} = Pr \text{ (Prandtl Number)}$$

By following a similar procedure, we can obtain

$$\pi_3 = \frac{hD}{k} = Nu \text{ (Nusselt Number)}$$

We may now express Eqn. (6.96)

$$F\left(\pi_1, \pi_2, \pi_3\right) \text{ as}$$

$$Nu = \phi\left(Re, Pr\right) \tag{6.97}$$

It is worthwhile to point out here that we chose the core variables quite arbitrarily. Had we chosen a different core group in our dimensional analysis, viz., D, ρ, μ, C_p the π group obtained would have been *Re*, *Pr* and a non-dimensional form of heat transfer coefficient which is designated as Stanton number St, and is expressed as

$$St = \frac{Nu}{Re.Pr} = \frac{h}{\rho U C_p}$$

So another form of correlating heat transfer data is

$$St = \phi(Re, Pr) \tag{6.98}$$

Dimensional analysis has thus shown us a way to reduce the seven significant variables of forced convection to three dimensionless parameters. We must now have experimental data in order to determine the functional relationship among these parameters.

A 30 cm long glass plate is hung vertically in the air at 27°C while its temperature is maintained at 77°C. Calculate the boundary layer thickness at the trailing edge of the plate.

If a similar plate is placed in a wind tunnel and air is blown over it at a velocity of 4 m/s, estimate the boundary layer thickness at its trailing edge.

Solution

Film temperature $T_f = (77 + 27)/2 = 52°C$

The properties of air at 52°C are; $k = 28.15 \times 10^{-3}$ W/mK

$$\nu = 18.41 \times 10^{-6}\ \text{m}^2/\text{s},\ Pr = 0.7,\ \beta = 3.07 \times 10^{-3}\ \text{K}^{-1}$$

(i) Free Convection

$$Gr_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{(9.81)\,(3.07 \times 10^{-3})\,(77 - 27)\,(0.3)^3}{(18.41 \times 10^{-6})^2}$$

$$= 1.2 \times 10^8$$

$\therefore$ *Rayleigh number*, $Ra_L = Gr_L \cdot Pr = 8.4 \times 10^7$

This value of the Rayleigh number, according to Eqn. (8.42), indicates a laminar boundary layer. The thickness of the boundary layer at the trailing edge is obtained from Eqn. (8.35) by putting $x = 0.3$

$$\delta_L = x[3.93\, Pr^{-1/2}(0.952 + Pr)^{1/4}\, Gr_x^{-1/4}]$$

$$= 0.3[3.93(0.7)^{-1/2}(0.952 + 0.7)^{1/4}(1.2 \times 10^8)^{-1/4}]$$

$$= 0.0152 \text{ m} = 1.52 \text{ cm}$$

(ii) Forced Convection

For air flow with $u_\infty = 4$ m/s

$$Re_L = \frac{u_\infty L}{\nu} = \frac{(4)\,(0.3)}{(18.41\times10^{-6})} = 6.51\times10^4$$

So the boundary layer is laminar. The boundary layer thickness at the trailing edge is given by Eqn. (7.13)

$$\delta_L = \frac{5L}{\sqrt{Re_L}} = \frac{(5)\,(0.3)}{(6.51\times10^4)^{\frac{1}{2}}} = 0.0058 = 0.58 \text{ cm} = 5.8 \text{ mm}$$

Thus the boundary layer thickness in forced convection is less than that in free convection.

III - UNIT

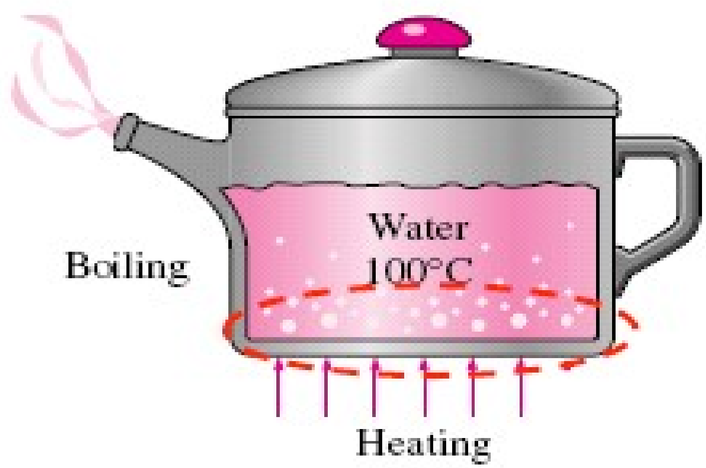
Boiling and Condensation

Boiling Heat Transfer Phenomenon

- **Boiling** is a liquid to vapor change process just like evaporation.
- Boiling is a phenomenon that occurs at a solid-liquid interface when a liquid is brought in contact with a surface maintained at a temperature sufficiently above the saturation temperature of the liquid.
- As the heat is conducted to the liquid vapor interface, bubbles are created by the expansion of entrapped gas or vapor at small cavities in the surface.

- The bubbles grow to a certain size, depending on the surface tension at the liquid-vapor interface and temperature and pressure.
- Boiling heat transfer is heat transferred by the boiling of water.
- Heat Transfer, $Q = h\ (T_s - T_{sat})$

where T_{sat} is the saturation temperature of the liquid.



Classification of Boiling

- Pool Boiling
- Flow Boiling
- Sub cooled Boiling
- Saturated Boiling

- **<u>Pool Boiling:</u>**

✓ Boiling is called pool boiling when bulk fluid motion is **absence**.

✓ Fluid motion is due to natural convection and bubble-induced mixing.

- **<u>Flow Boiling:</u>**

✓ Boiling in the **presence** of bulk fluid motion is called flow boiling (Forced Convection Boiling).

✓ Fluid motion is induced by external means such as pump, as well as by bubble-induced mixing.

Sub cooled Boiling:

- ❖ When the temperature of the liquid is below the saturation temperature.
- ❖ The term **sub cooling** refers to a liquid existing at a temperature **below** its normal boiling point.

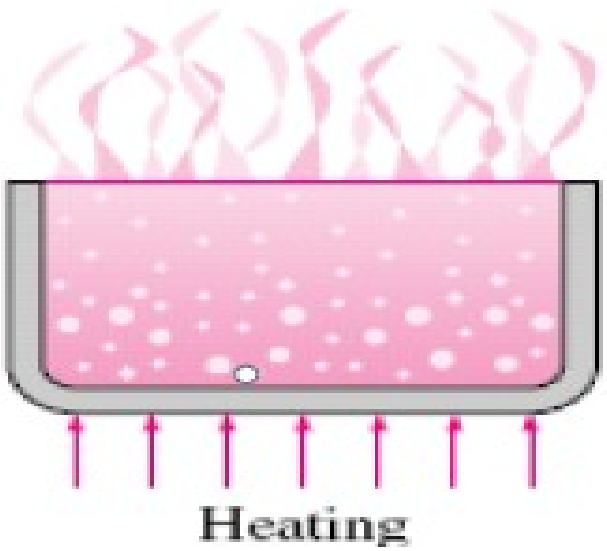
Saturated Boiling:

- ❖ When the temperature of the liquid is equal to the saturation temperature.

➢ Sub cooled and saturated boiling can exist in both nucleate and film boiling.

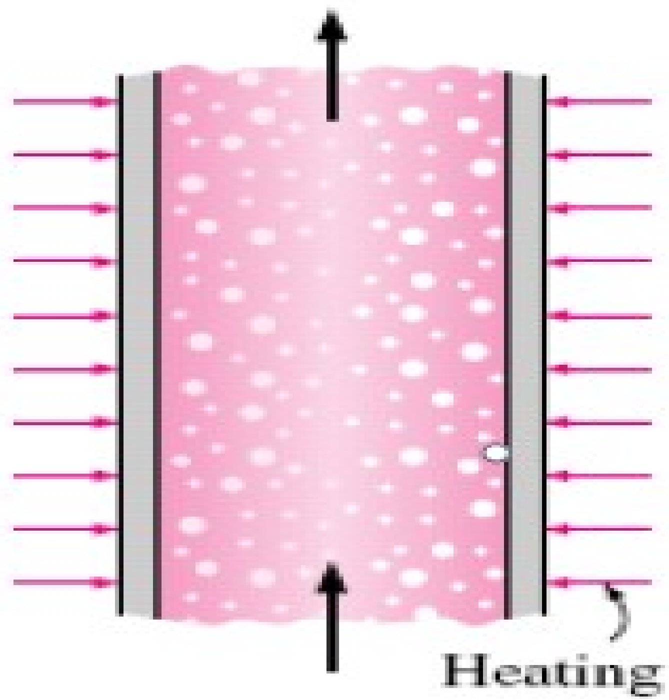
Pool Boiling

- Boiling is called **pool boiling** in the absence of bulk fluid flow.
- Any motion of the fluid is due to natural convection currents and the motion of the bubbles under the influence of buoyancy.



Flow Boiling

- Boiling is called **flow boiling** in the presence of bulk fluid flow.
- In flow boiling, the fluid is forced to move in a heated pipe or over a surface by external means such as a pump.

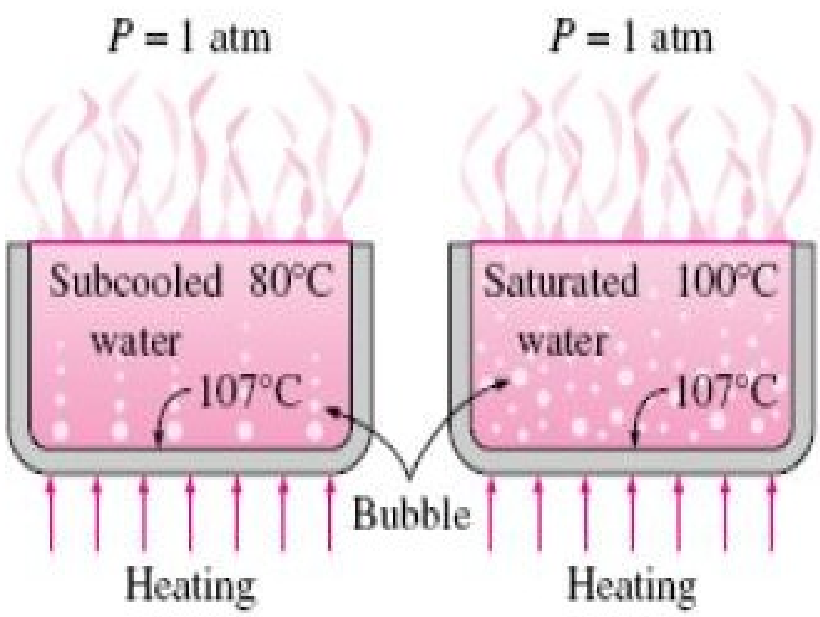


Subcooled Boiling

- When the temperature of the main body of the liquid is below the saturation temperature.

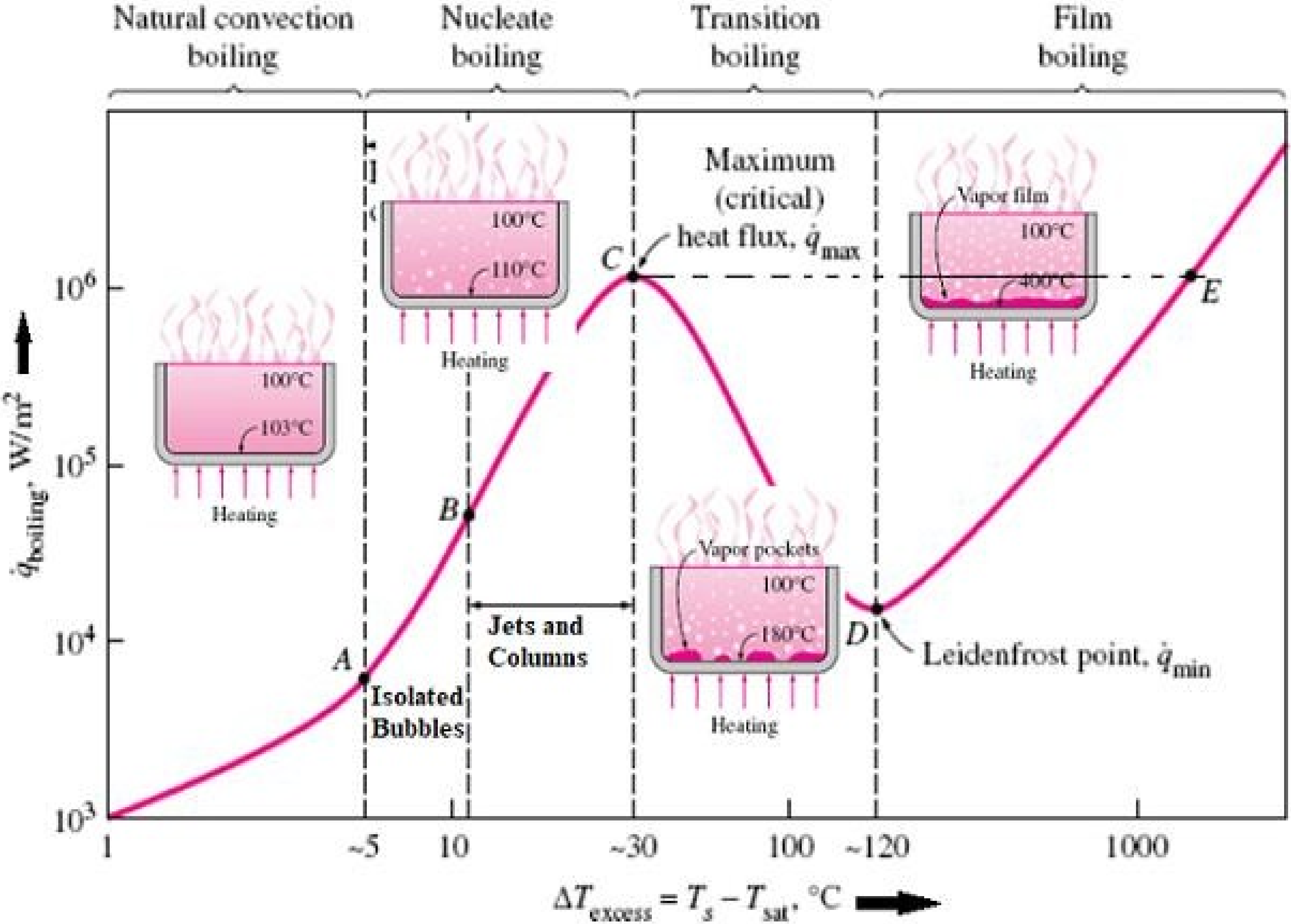
Saturated Boiling

- When the temperature of the liquid is equal to the saturation temperature.



The Boiling Curve

- In a typical boiling curve, four different boiling regimes are observed: **natural convection boiling, nucleate boiling, transition boiling, and film boiling** depending on the excess temperature $\Delta T_{excess} = T_s - T_{sat}$.


Natural convection boiling
Nucleate boiling
Transition boiling
Film boiling
Maximum (critical) heat flux, $\dot{q}_{max}$
C
100°C
110°C
Heating
100°C
103°C
Heating
Vapor film
100°C
400°C
Heating
E
B
A
Isolated Bubbles
Jets and Columns
Vapor pockets
100°C
180°C
Heating
D
Leidenfrost point, $\dot{q}_{min}$
10^6
10^5
10^4
10^3
$\dot{q}_{boiling}$, W/m²
1
~5
10
~30
100
~120
1000
$\Delta T_{excess} = T_s - T_{sat}$, °C

Natural Convection Boiling (to Point A)

- Liquid is slightly superheated in this case (a metastable condition) and evaporates when it rises to the free surface.
- Liquid motion is due to n**atural convection.**
- In region *1* called free convection zone ,the excess temperature is very small.
- Here the liquid near the surface is super heated slightly, the convection currents circulate the liquid and evaporation takes place at the liquid surface.

Nucleate Boiling (between Points A and C)

- Bubbles start forming at point A and increases number of nucleation sites as we move towards point C.
- **Region A–B** — isolated bubbles are formed and heat flux rise sharply with increasing ΔT_{excess}. This region is the beginning of nucleate boiling.
- **Region B–C** — Increasing number of nucleation sites causes bubble interactions and coalescence into jets and column. Heat flux increases at lower rate and **maximum at point C. T**he maximum heat flux known as critical heat flux occurs at point C.

- **Critical Heat Flux** - CHF, ($\Delta Te \approx 30ºC$) ¬ Maximum attainable heat flux in nucleate boiling.
- $q'' \approx 1$ MW/m for water at atmospheric pressure.
- Point C on the boiling curve is also called the **burnout point**, and the heat flux at this point the **burnout heat flux.**

Transition Boiling(between Points C and D)

•When ΔT_{excess} increases past point C, heat flux decreases because a large fraction of the heater surface is covered by a **vapor film**, which acts as an **insulation.**

•The transition boiling regime, which is also called the unstable film boiling regime.

Film Boiling (beyond Point D)

•At **point D**, where the heat flux reaches a **minimum** is called the **Leidenfrost point.**

•Heat transfer is by conduction and radiation across the vapor blanket, therefore, heat transfer rate increases with increasing excess temperature.

•The Leidenfrost effect is a physical phenomenon in which a liquid, close to a surface that is significantly hotter than the liquid's boiling point.

- The phenomenon of stable film boiling can be observed when a drop of water falls on a red hot stove. The drop does not evaporate immediately but moves a few times on the stove.

Condensation

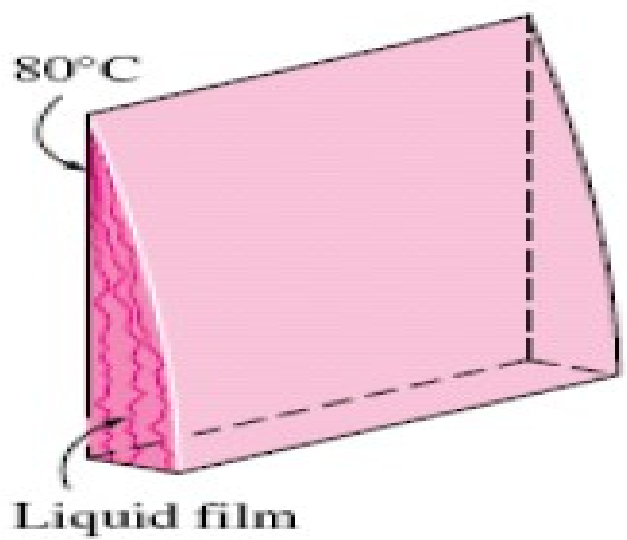
- The process of condensation is the reverse of boiling.
- Condensation occurs when the temperature of a vapor is reduced **below** its saturation temperature.

Two forms of condensation:

- – *Film condensation,*
- – *Drop wise condensation.*

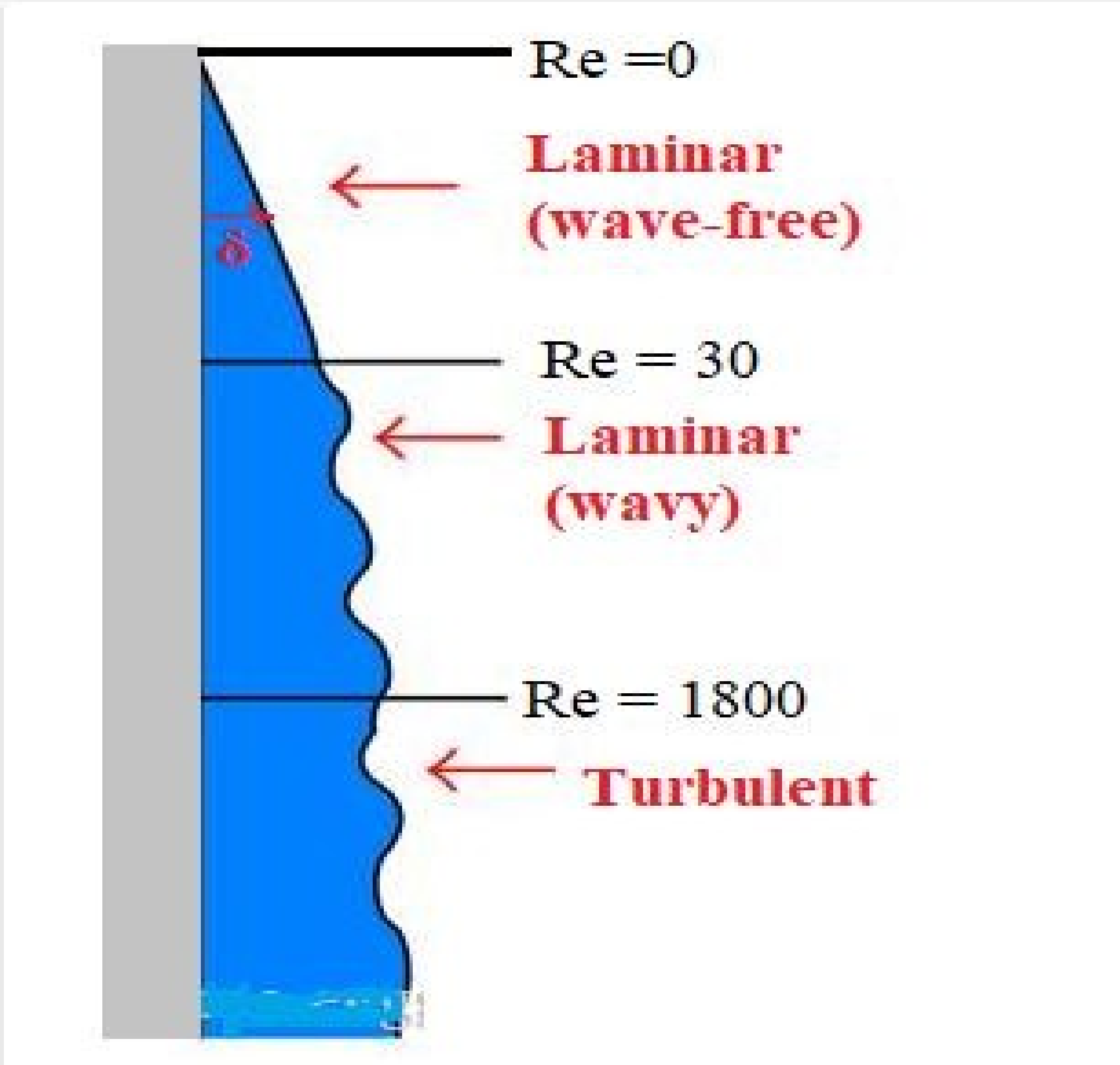
Film condensation

- The condensate wets the surface and forms a liquid film.
- The surface is blanketed by a liquid film which serves as a *resistance* to heat transfer.



- Film wise condensation generally occurs on clean uncontaminated surfaces.
- In this type of condensation the film covering the entire surface grows in thickness as it moves down the surface by gravity.
- There exists a thermal gradient in the film and so it acts as a resistance to heat transfer.

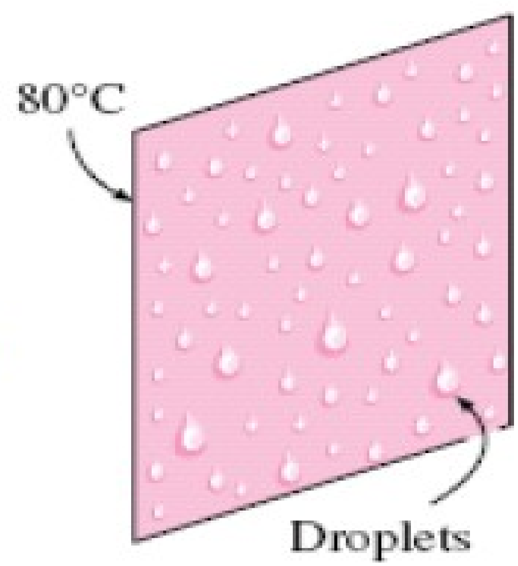
- **$Re_f = d_H \rho V / (\mu)$**
- d_H = Hydraulic diameter
- ρ = density of liquid
- V = average velocity of flow
- μ = viscosity of fluid



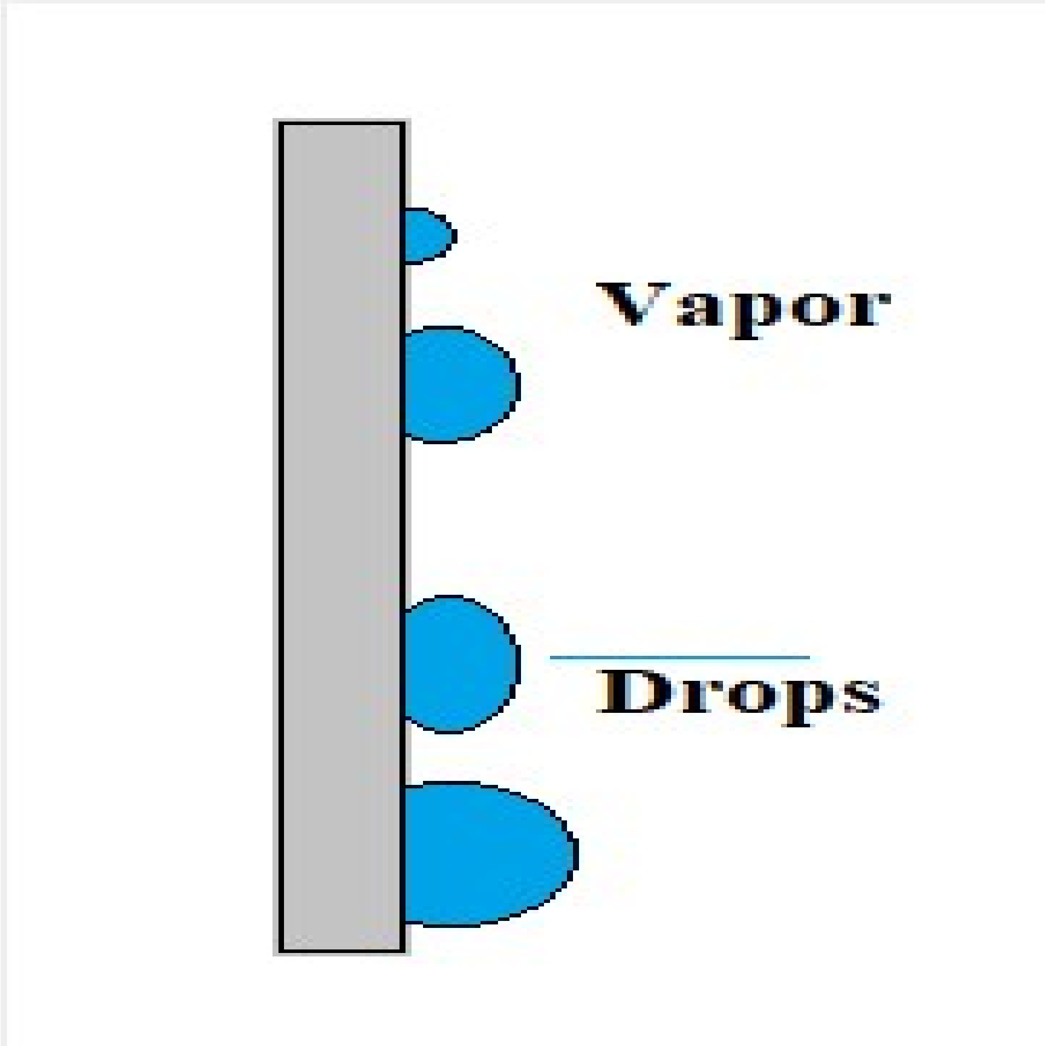
Re =0
Laminar
(wave-free)
δ
Re = 30
Laminar
(wavy)
Re = 1800
Turbulent

Dropwise condensation

- The condensed vapor forms droplets on the surface.
- The droplets slide down when they reach a certain size.
- No liquid film to resist heat transfer.
- As a result, heat transfer rates that are more than 10 times larger than with film condensation can be achieved.



- In drop wise condensation the vapur condenses into small liquid droplets of various sizes which fall down the surface in a random fashion.
- A large portion of the plate is directly exposed to the vapor, making heat transfer rates much larger than those in film condensation (5 to 10 times).


Vapor
Drops

- Drop wise condensation is achieved by-
- Adding a promoting chemical into the vapor **(wax, fatty acid),**
- Treating the surface with a promoter chemical,
- Coating the surface with a polymer such as teflon or a noble metal such as Au, Ag, Rh, Pd, Pt

Comparison between film condensation and dropwise condensation

Film Condensation	Dropwise Condensation
1. In *film condensation*, the condensate wets the surface and forms a liquid film on the surface that slides down under the influence of gravity.	**1.** In *dropwise condensation*, the condensed vapour forms countless droplets of varying diameters on the surface instead of a continuous film.
2. Relatively less heat-transfer coefficients are associated with film condensation.	**2.** Higher heat-transfer coefficients (about 5.10 times greater than those in film condensation) can be achieved.

3. On a rusty or etched plate, the vapour is condensed in a continuous film over the entire wall	**3.** With a polished surface, the condensate is formed in drops which rapidly grow in size (up to 3 mm in diameter) and roll down the surface.
4. The condensate itself forms a film (layer) on the surface which imposes some extra thermal resistance.	**4.** Droplets provide very little thermal resistance.

IV - UNIT

HEAT EXCHANGERS

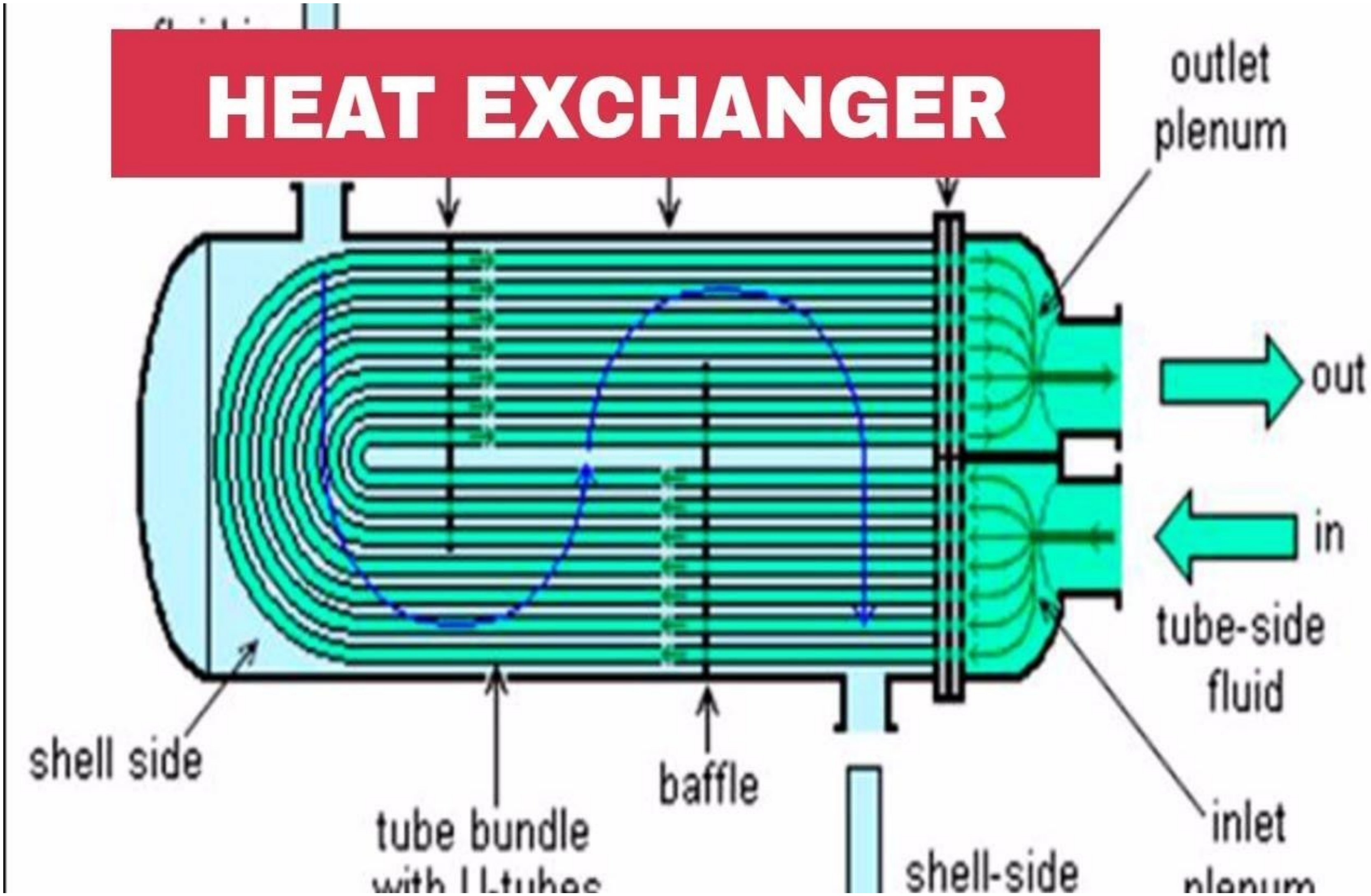
HEAT EXCHANGERS

- A heat exchanger is a system used to transfer heat between two or more fluids.
- Heat exchangers are used in both cooling and heating processes.
- The fluids may be separated by a solid wall to prevent mixing or they may be in direct contact.
- They are widely used in space heating, refrigeration, air conditioning, power stations, chemical plants, petrochemical plants, petroleum refineries, natural-gas processing, and sewage treatment.

Shell and Tube;

Plate Type;

Shell and Tube HE



One pass shell and tube HE

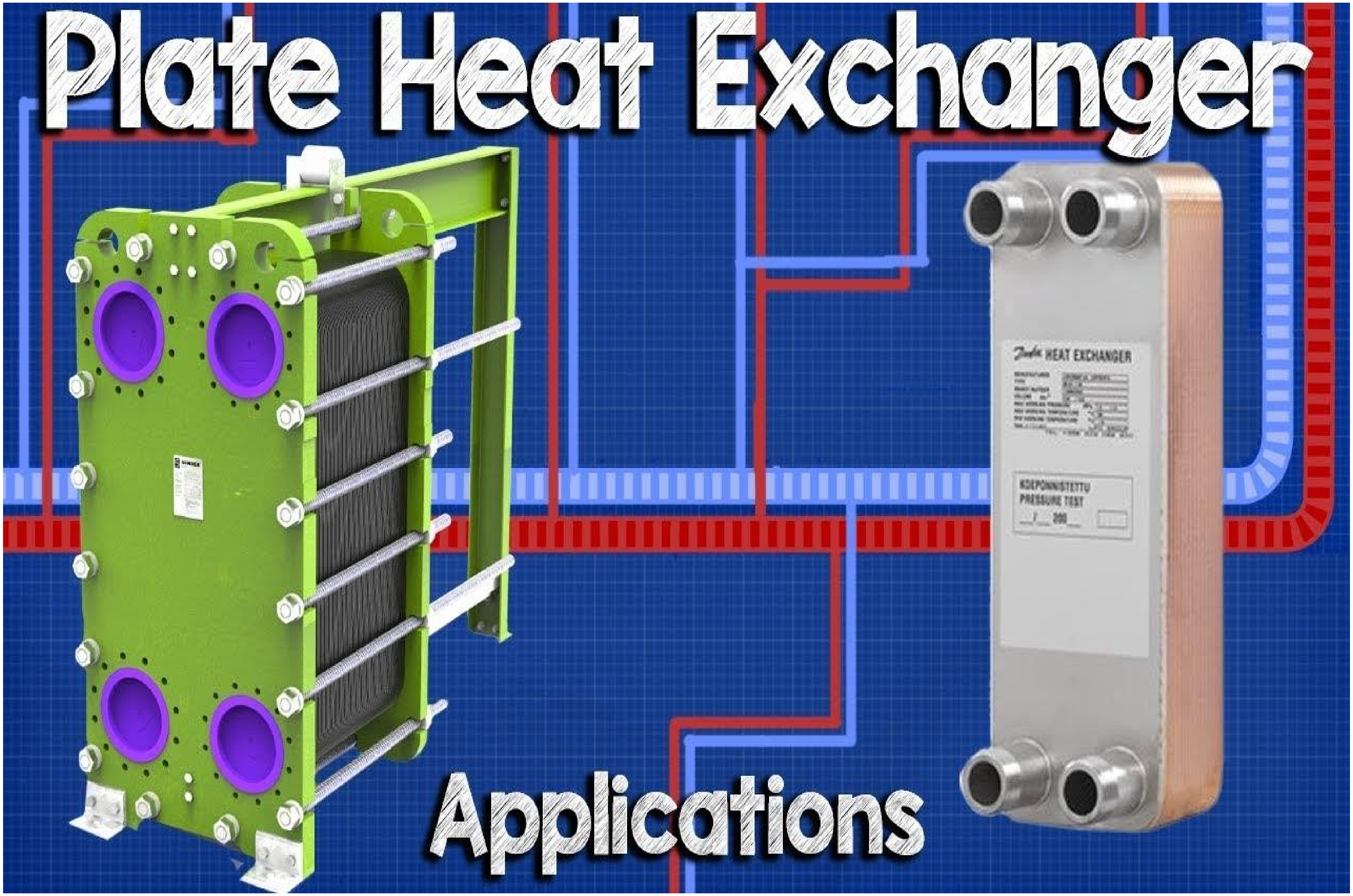
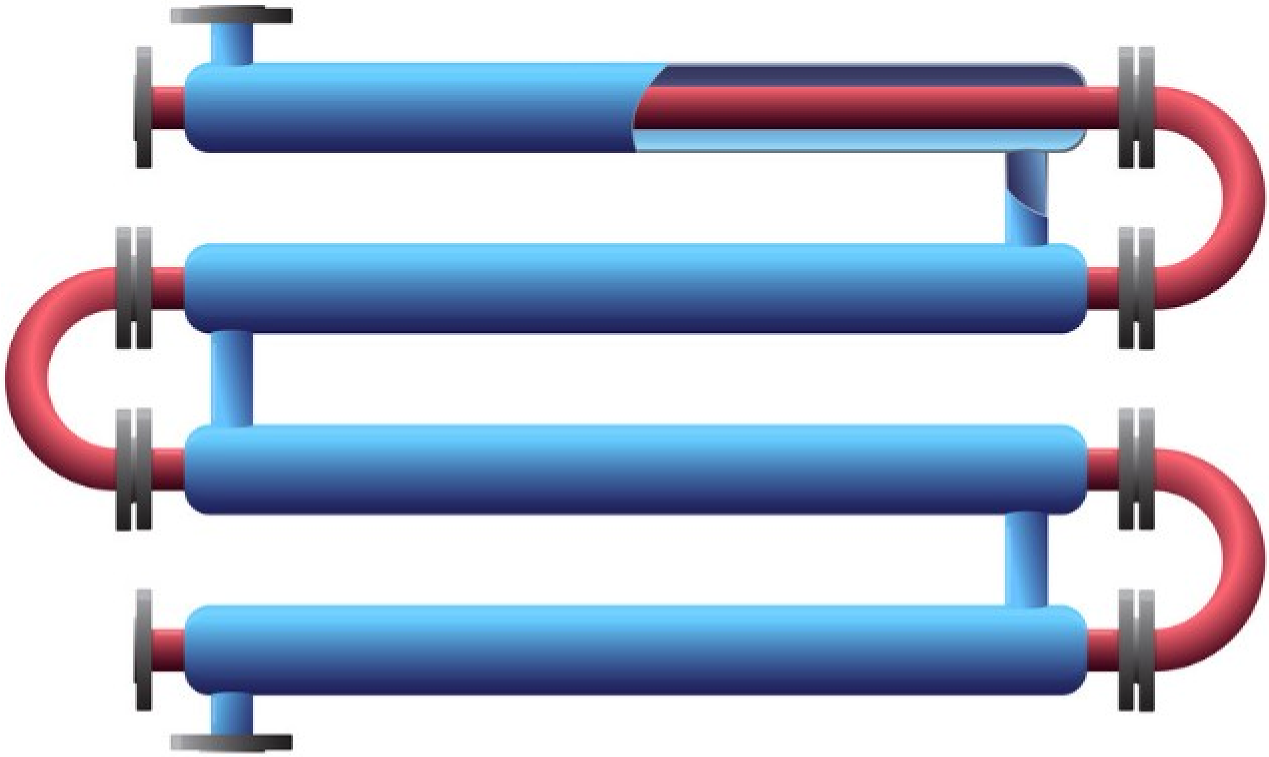


Plate Heat Exchanger
Applications
HEAT EXCHANGER
PRESSURE TEST

Two pass HE



The thermal design of a heat exchanger involves the calculation of the necessary surface area required to transfer heat at a given rate for given flow rates and fluid temperatures. The concept of overall heat transfer coefficient, U, introduced in Section 1.9, is of great significance in the heat exchanger calculations. As defined in Eqn. (1.27)

$$Q = UA\,\Delta T_m \tag{12.1}$$

where ΔT_m is an average effective temperature difference for the entire heat exchanger.

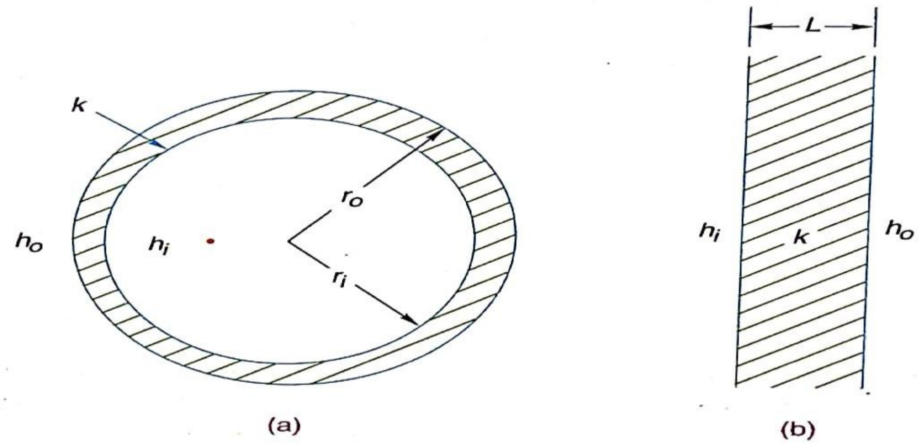


Fig. 12.5 Heat Exchanger Walls: (*a*) Cylindrical, (*b*) Plane

Recall from Eqn. (1.29) that the overall heat transfer coefficient is defined in terms of the total resistance. For the common configurations, plane and cylindrical walls of Fig. 12.5, this coefficient is of the form

Plane wall:
$$U = \frac{1}{1/h_o + L/k + 1/h_i} \tag{12.2}$$

Cylindrical wall:
$$U_o = \frac{1}{\dfrac{1}{h_o} + \dfrac{r_o}{k}\,\text{In}\left(\dfrac{r_o}{r_i}\right) + \left(\dfrac{r_o}{r_i}\right)\dfrac{1}{h_i}} \tag{12.3}$$

or
$$U_i = \frac{1}{\dfrac{1}{h_i} + \dfrac{r_i}{k}\,\text{In}\left(\dfrac{r_o}{r_i}\right) + \left(\dfrac{r_i}{r_o}\right)\dfrac{1}{h_o}} \tag{12.4}$$

where i and o represent the inside and outside surfaces of the wall, respectively.

Since the surface areas for heat transfer on the inner and outer surfaces are not the same, so we have two overall coefficients as defined above. However, for the sake of compatibility

Example 12.1

Water heated to 80°C flows through a 2.54 cm I.D. and 2.88 cm O.D. steel $(k = 50\ \text{W/m K})$ tube. The tube is exposed to an environment which is known to provide an average convection coefficient of $h_o = 30800\ \text{W/m}^2\text{K}$ on the out side of the tube. The water velocity is 50 cm/s. Calculate the overall heat transfer coefficient, based on the outer area of the pipe.

Solution

The properties of water at the bulk temperature of 80°C are

$$\rho = 974\ \text{kg/m}^3, \quad \nu = 0.364 \times 10^{-6}\ \text{m}^2/\text{s}$$
$$k = 668.7 \times 10^{-3}\ \text{W/mK}, \quad Pr = 2.20$$

The Reynolds number is

$$Re_D = \frac{UD}{\nu} = \frac{(0.50)\ (0.0254)}{0.364 \times 10^{-6}} = 34890$$

Accordingly, the flow is turbulent and the convective coefficient may be calculated from Eqn. (7.108)

$$Nu_D = 0.023\ Re_D^{4/5}\ Pr^{0.4}$$
$$= (0.023)\ (34890)^{0.8}\ (2.20)^{0.4}$$
$$= 135.79$$

Hence

$$h_i = Nu_D \frac{k}{D_i}$$

$$= \frac{(135.79)\ (668.7 \times 10^{-3})}{(0.0254)} = 3575\ \text{W/m}^2\ \text{K}$$

h_o (given) = 30800 W/m^{2}K

The overall heat transfer coefficient may now be computed by Eqn. (12.3)

$$U_o = \frac{1}{\frac{1}{h_o} + \frac{r_o}{k}\ln\left(\frac{r_o}{r_i}\right) + \left(\frac{r_o}{r_i}\right)\frac{1}{h_i}}$$

$$= \frac{1}{\frac{1}{(30800)} + \frac{0.0144}{50}\ln\left(\frac{2.88}{2.54}\right) + \left(\frac{2.88}{2.54}\right)\left(\frac{1}{3575}\right)}$$

$= 2591.9$ W/m^{2}K.

12.4 FOULING FACTORS

Equation (12.2) through Eqn. (12.4) are, in fact, valid only for clean surfaces. However, it is a well-known fact that the surfaces of a heat exchanger do not remain clean after it has been in use for some time. The surfaces become fouled with scalings or deposits which are formed due to impurities in the fluid, chemical reaction between the fluid and the wall material, rust formation, etc. The effect of these deposits is felt in terms of greatly increased surface resistance affecting the value of U. This effect is taken care of by introducing an additional thermal resistance called the *fouling resistance* $R_f \cdot R_f$ must be determined experimentally by testing the heat exchanger in both clean and dirty conditions, being defined by

$$\frac{1}{U_{\text{foul}}} = R_f + \frac{1}{U_{\text{clean}}} \tag{12.6}$$

Denoting the fouling resistance by R_{f_i} and R_{f_o} at the inner and outer surfaces, respectively, Eqns. (12.3) and (12.4) stand modified to

$$U_o = \frac{1}{\dfrac{1}{h_o} + R_{f_o} \dfrac{r_o}{k} \ln\left(\dfrac{r_o}{r_i}\right) + \left(\dfrac{r_o}{r_i}\right) R_{f_i} + \left(\dfrac{r_o}{r_i}\right)\dfrac{1}{h_i}} \tag{12.7}$$

and

$$U_i = \frac{1}{\dfrac{1}{h_i} + R_{f_i} \dfrac{r_i}{k} \ln\left(\dfrac{r_o}{r_i}\right) + \left(\dfrac{r_i}{r_o}\right) R_{f_o} + \left(\dfrac{r_i}{r_o}\right)\dfrac{1}{h_o}} \tag{12.8}$$

Some typical values of R_f are given in Table 12.2

Example 12.2

Determine the overall heat transfer coefficient U_0 based on the outer surface of a 2.54 cm O.D., 2.286 cm I.D. heat exchanger tube ($k = 102$ W/m K), if the heat transfer coefficients at the inside and outside of the tube are $h_i = 5500 \text{ W/m}^2\text{K}$ and $h_o = 3800 \text{ W/m}^2\text{K}$ respectively and the fouling factors are $R_{f_o} = R_{f_i} = 0.0002 \text{ m}^2 \text{k/W}$

Solution

Using Eqn. (12.7), the overall heat transfer coefficient based on the outside area of the tube becomes

$$U_o = \frac{1}{\frac{1}{h_o} + R_{f_o} \frac{r_o}{k} \ln\left(\frac{r_o}{r_i}\right) + \left(\frac{r_o}{r_i}\right) R_{f_i} + \left(\frac{r_o}{r_i}\right) \frac{1}{h_i}}$$

$$= \frac{1}{\frac{1}{3800} + 0.0002 \frac{(0.0127)}{102} \ln\left(\frac{2.54}{2.286}\right) + (0.0002)\left(\frac{2.54}{2.286}\right) + \left(\frac{2.54}{2.86}\right)\left(\frac{1}{5000}\right)}$$

$$= 1110 \text{ W/m}^2\text{K}.$$

12.5 LMTD METHOD OF HEAT EXCHANGER ANALYSIS

The thermal analysis of any heat exchanger involves variables like inlet and outlet fluid temperatures, the overall heat transfer coefficient, total surface area for heat transfer and the total heat transfer rate. Since the hot fluid is transferring a part of its energy to the cold fluid, there will be an increase in enthalpy of the cold fluid and a corresponding decrease in enthalpy of the hot fluid. This may be expressed as

$$Q = \dot{m}_h c_h (T_{h,i} - T_{h,o}) \tag{12.9}$$

and

$$Q = \dot{m}_c c_c (T_{c,o} - T_{c,i}) \tag{12.10}$$

where $\dot{m}$ = mass flow rate

c = constant pressure specific heat.

The subscripts c and h indicate the cold and hot fluids, whereas the subscripts i and o refer to the fluid inlet and outlet conditions, respectively.

If we denote the temperature difference between the hot and cold fluids by

$$\Delta T = T_h - T_c \tag{12.11}$$

since ΔT is varying with position in the heat exchanger, the actual rate equation for heat transfer will be Eqn. (12.1)

$$Q = UA\Delta T_m \tag{12.1}$$

where ΔT_m is a suitable mean temperature difference across the heat exchanger. This average or mean value must be determined before use can be made of Eqn. (12.1). We shall now present a method for the determination of the mean temperature difference. Since the final expression obtained by this method will be in the form of a logarithmic relation, this method is referred to as *Logarithmic Mean Temperature Difference (LMTD) method of analysis.*

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12.5.1 Parallel Flow Heat Exchanger

Let us first consider a parallel flow heat exchanger as depicted in Fig. 12.6. Assuming that:

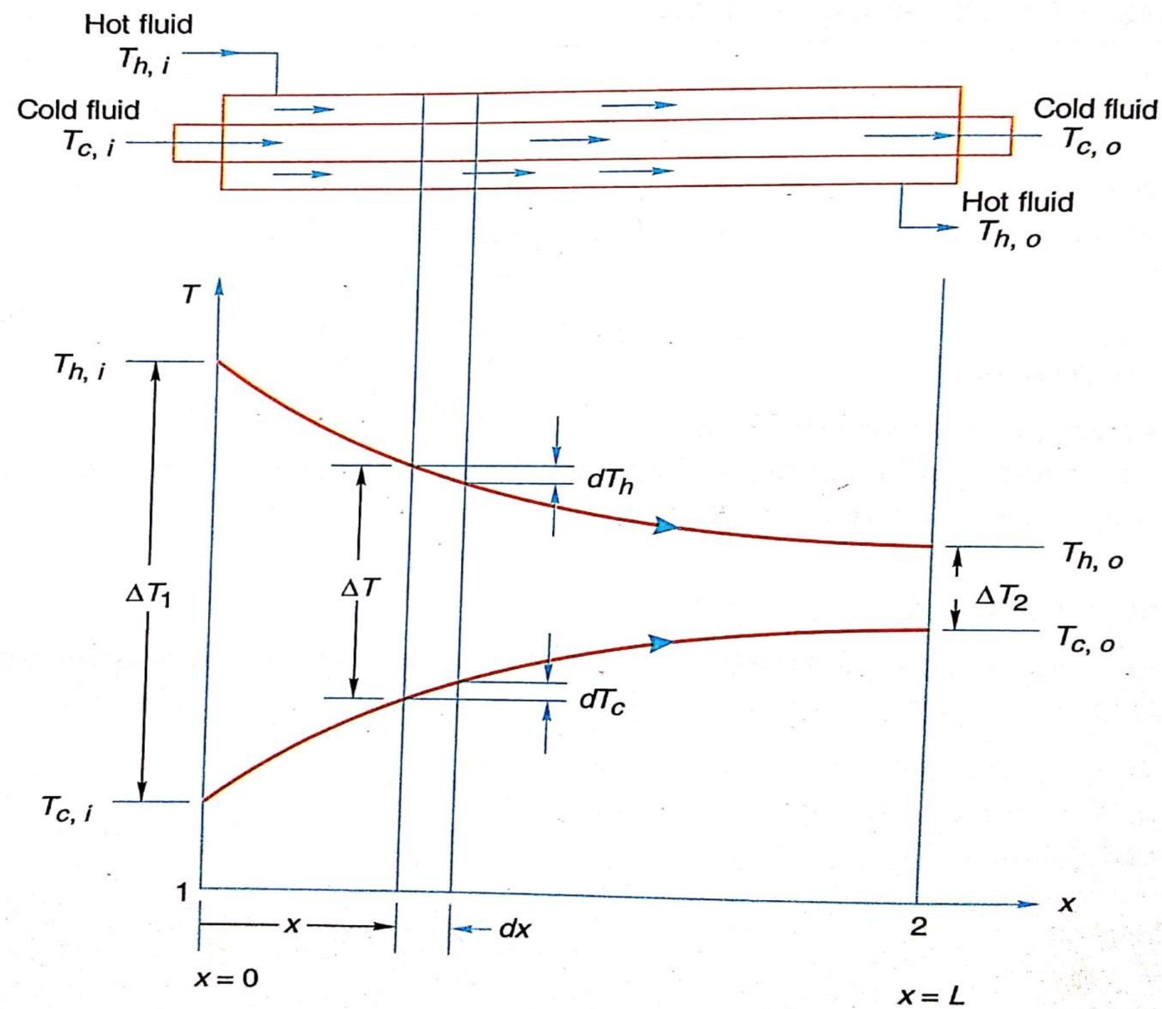


Fig. 12.6 Temperature Distribution for a Parallel Flow Heat Exchanger

or

$$Q = UA \frac{\Delta T_2 - \Delta T_1}{\text{In } (\Delta T_2 / \Delta T_1)} \quad (12.17)$$

where

$$\Delta T_1 = T_{h,i} - T_{c,i} \text{ (from Fig. 12.6)} \quad (12.18)$$

$$\Delta T_2 - T_{h,o} - T_{c,o}$$

On comparing this result with Eqn. (12.1), we see that the appropriate average temperature difference is a log mean temperature difference.

LMTD, ΔT_{lm}. So we may write

$$Q = UA\,\Delta T_{lm} \quad (12.19)$$

where

$$\Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\text{In } (\Delta T_2 / \Delta T_1)}$$

or

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\text{In } (\Delta T_1 / \Delta T_2)} \quad (12.20)$$

12.5.2 Counter Flow Heat Exchanger

A counter flow heat exchanger, where the fluids, move in parallel but opposite directions, is shown in Fig. 12.7. The change in temperature difference between the two fluids is greatest at the entrance of a parallel flow heat exchanger but it may not be so in a counter flow arrangement.

The analysis of a counter flow heat exchanger can be done exactly in the same manner as outlined in the previous section for a parallel flow exchanger. Eqns. (12.1), (12.9) and (12.10) are, in fact, valid for any heat exchanger. By taking a differential area element for a counter flow exchanger (Fig. 12.7) and proceeding as before it can be easily shown that Eqns. (12.19) and (12.20) are valid in this case too.

Hence

$$Q = UA\Delta T_{lm} = UA\frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} \tag{12.21}$$

where

$$\Delta T_1 = T_{h,i} - T_{c,o}$$

$$\Delta T_2 = T_{h,o} - T_{c,i}$$

Referring to Fig. 12.7

$$\Delta T_1 = T_{h,i} - T_{c,o} = 80 - 43.8 = 36.2$$

$$\Delta T_2 = T_{h,o} - T_{c,i} = 50 - 25 = 25$$

Using Eqn. (12.17)

$$Q = UA \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2/\Delta T_1)}$$

where

$$Q = \left(\frac{10000}{3600}\right)(2095)(80-50) = 174583.33 \text{ W}$$

∴

$$A = \frac{Q}{U}\frac{\ln(\Delta T_2/\Delta T_1)}{\Delta T_2 - \Delta T_1} = \frac{174583.33}{300}\frac{\ln\left(\frac{25}{36.2}\right)}{(25 - 36.2)}$$

$$= 19.23 \text{ m}^2.$$

Example 12.4

Hot oil with a capacity rate of 2500 W/K flows through a double pipe heat exchanger. It enters at 360°C and leaves at 300°C. Cold fluid enters at 30°C and leaves at 200°C. If the overall heat transfer coefficient is $800\ \text{W/m}^2\text{K}$, determine the heat exchanger area required for

(*a*) parallel flow and (*b*) counter flow.

Solution

The heat transfer from the oil is

$$Q = C_h \Delta T = 2500(360 - 300) = 150\text{kW}$$

(*a*) The temperature distribution in a parallel flow heat exchanger is as shown in Fig. 12.6. The LMTD is given by Eqn. (12.20).

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\text{In}\ (\Delta T_1/\Delta T_2)}$$

for which

$$\Delta T_1 = T_{h,i} - T_{c,i} = 360 - 30 = 330°\text{C}$$

$$\Delta T_2 = T_{h,o} - T_{c,o} = 300 - 200 = 100°\text{C}$$

$$\Delta T_{lm} = \frac{330 - 100}{\text{In}\ (330/100)} = 192.64°\text{C}$$

The heat exchanger area may be calculated from Eqn. (12.19).

$$A = \frac{Q}{U\,\Delta T_{lm}} = \frac{(150000)}{(800)(192.64)} = 0.973\ \text{m}^2$$

(*b*) Figure 12.7 is a qualitative representation of the temperature distribution in a counter flow case.

Here

$$\Delta T_1 = T_{h,i} - T_{c,i} = 360 - 200 = 160°\text{C}$$

$$\Delta T_2 = T_{h,o} - T_{c,i} = 300 - 30 = 270°\text{C}$$

Equation (12.20) yields

$$\Delta T_{lm} = \frac{160 - 270}{\text{In }(160/270)} = 210.22°\text{C}$$

Equation (12.19) gives

$$A = \frac{Q}{U\,\Delta T_{lm}} = \frac{(150000)}{(800)\,(210.22)} = 0.892 \text{ m}^2$$

Thus we see that for the same terminal temperatures of fluids, the surface area required for a count flow arrangement is less than that in a parallel flow arrangement.

12.5.3 Condensers and Evaporators

Two special forms of heat exchangers, namely condensers and evaporators, are employed in many industrial applications. One of the fluids flowing through these exchangers changes phase. The temperature distributions in these exchangers are shown in Fig. 12.8. In the case of a condenser, the hot fluid will remain at a constant temperature, provided its pressure does not change, while the temperature of the cold fluid increases. This is possible only when $C_h \gg C_c$, in fact, $C_h \to \infty$. In the case of an evaporator $C_h \ll C_c$ or $C_c \to \infty$, the cold fluid temperature remains uniform and it undergoes a phase change.

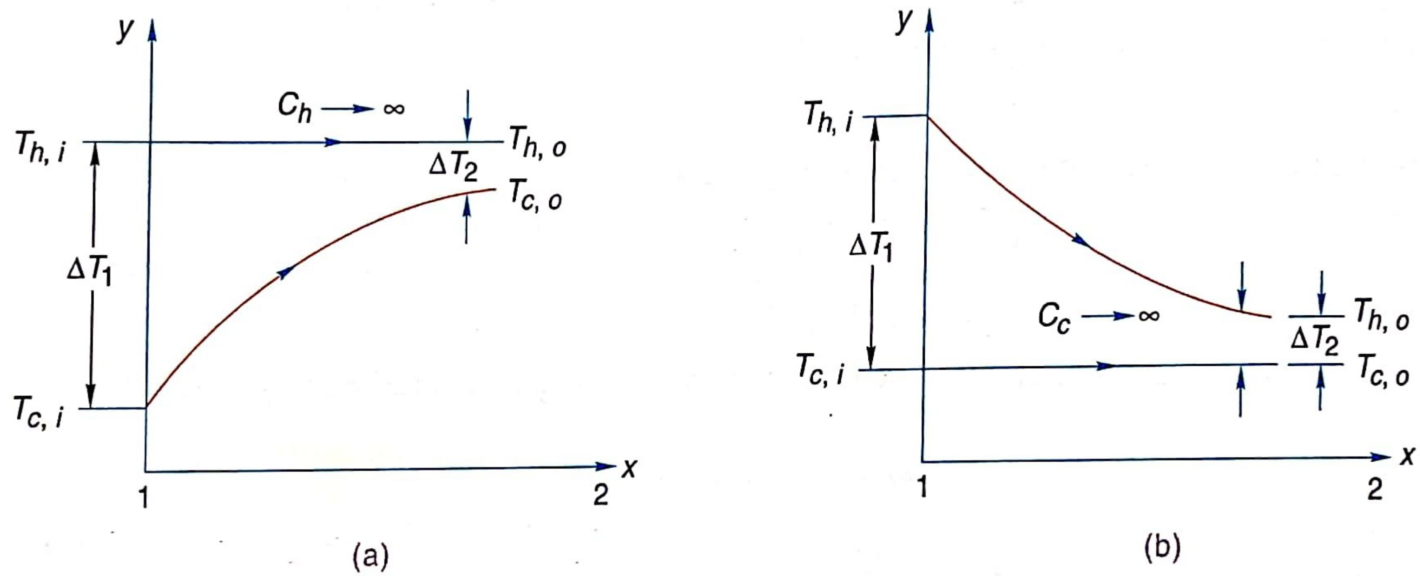


Fig. 12.8 Temperature Distribution of Fluids in (*a*) Condenser (*b*) Evaporator

It is interesting to note that in a phase change process it is immaterial whether we have parallel flow, counter flow or cross flow arrangements. Use of Eqn. (12.21) can still be made of in these exchangers.

Example 12.6

Saturated steam at 120°C is condensing on the outer tube surface of a single pass heat exchanger. The heat transfer coefficient is $U_0 = 1800\ \text{W/m}^2\text{K}$. Determine the surface area of a heat exchanger capable of heating 1000 kg/h of water from 20°C to 90°C. Also compute the rate of condensation of steam

$$h_{fg} = 2200\ \text{kJ/kg}.$$

Solution

The temperature distribution in a condenser is shown in Fig. 12.8 (*a*)

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{(T_{h,i} - T_{c,i}) - (T_{h,i} - T_{c,o})}{\ln\left[(T_{h,i} - T_{c,i})/(T_{h,i} - T_{c,o})\right]}$$

$$= \frac{(120 - 20) - (120 - 90)}{\ln [(120 - 20)/(120 - 90)]}$$

$$= \frac{100 - 30}{\ln (10/3)} = \frac{70}{1.204} = 58.14°C$$

The rate of heat transfer

$$Q = \dot{m}_c \, c_c \, (T_{c,o} - T_{c,i})$$

$$= \left(\frac{1000}{3600}\right)(4186)(90 - 20)$$

$$= 81394.4 \text{ W}$$

Also $$Q = UA \, \Delta T_{lm}$$

∴ $$A = \frac{Q}{U \, \Delta T_{lm}} = \frac{81394.4}{(1800)(58.14)} = 0.78 \text{ m}^2$$

$$Q = \dot{m}_s \, h_{fg}$$

$$\dot{m}_s = \frac{Q}{h_{fg}} = \frac{81394.4}{(1000)(2200)} = 0.037 \text{ kg/s}$$

$$= 133.2 \text{ kg/h}.$$

12.5.4 Multiple-pass and Cross Flow Heat Exchangers

The flow conditions in multiple-pass and cross flow heat exchangers are much more complicated than those in concentric tube, single pass heat exchangers. For these complex situations, the determination of the mean effective temperature difference is so difficult that the usual practice is to modify Eqn. (12.19) by a correction factor, F, giving

$$Q = UA\,(F\Delta T_{lm}) \tag{12.24}$$

Wherein ΔT_{lm} is the LMTD for a counter flow double pipe arrangement with the same hot and cold fluid temperatures as in the more complex design. Expressions for the correction factor, F, for various cross flow and shell-and-tube designs have been developed (see *STEM A*: 1978; Kern; 1957; Jakob: 1957). A more convenient way of representing these correction factors is the chart or graphical form. Correction factors for several different types of heat exchangers are given in Figs. 12.9 through 12.12, according to Incropera and Dewitt (1981). In these figures, the notation (T, t) has been used to specify the fluid temperature, t being used for the tube fluid and T for the shell fluid. The two temperature ratios, P and R are defined as

$$R = \frac{T_i - T_o}{t_o - t_i}$$

$$P = \frac{t_o - t_i}{T_i - t_i}$$

A good design should involve the selection of parameters P and R such that the value of F is always greater than 0.75.

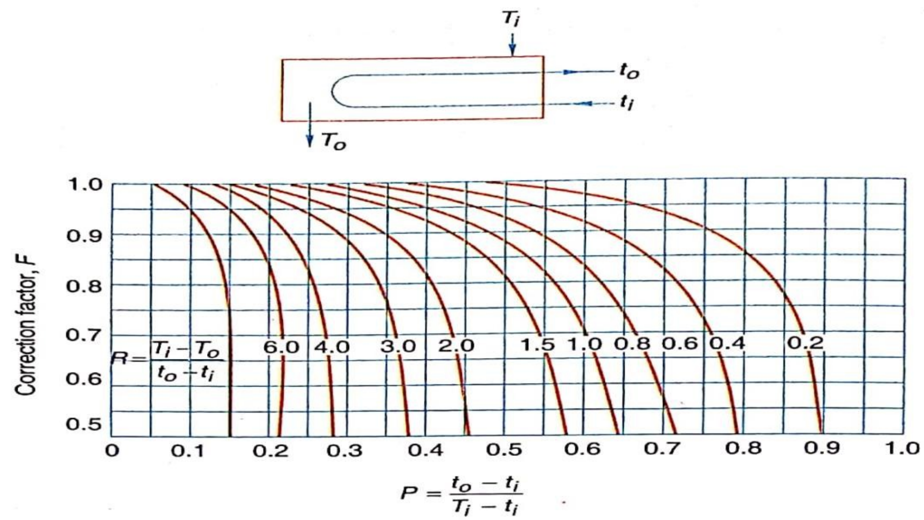


Fig. 12.9 Correction Factor Plot for a Shell-and-Tube Heat Exchanger with One Shell Pass and Two, Four or Multiple Tube Passes

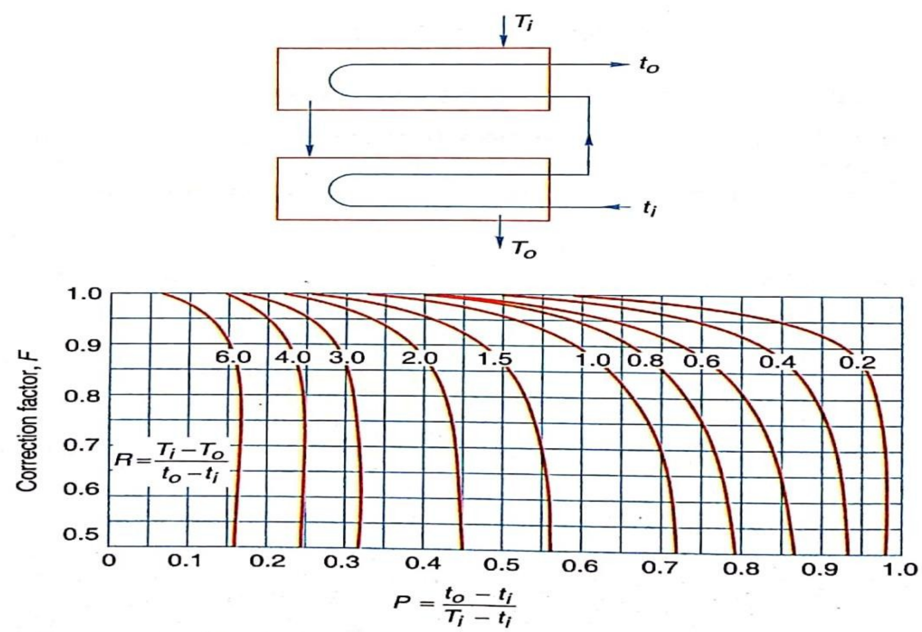


Fig. 12.10 Correction Factor Plot for a Shell-and-Tube Heat Exchanger with Two Shell Passes and Four, Eight or any Multiple of Four Tube Passes

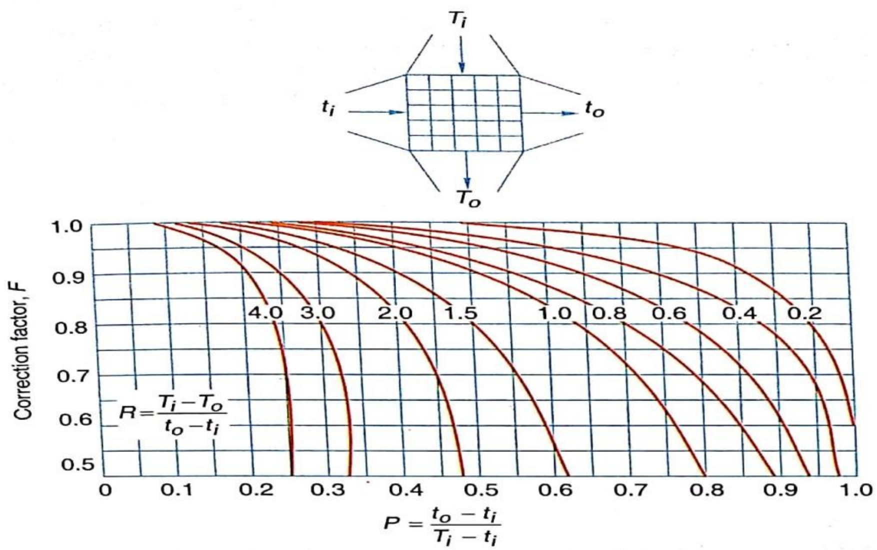


Fig. 12.11 Correction Factor Plot for Single Pass Cross Flow Heat Exchanger, both Fluids Unmixed

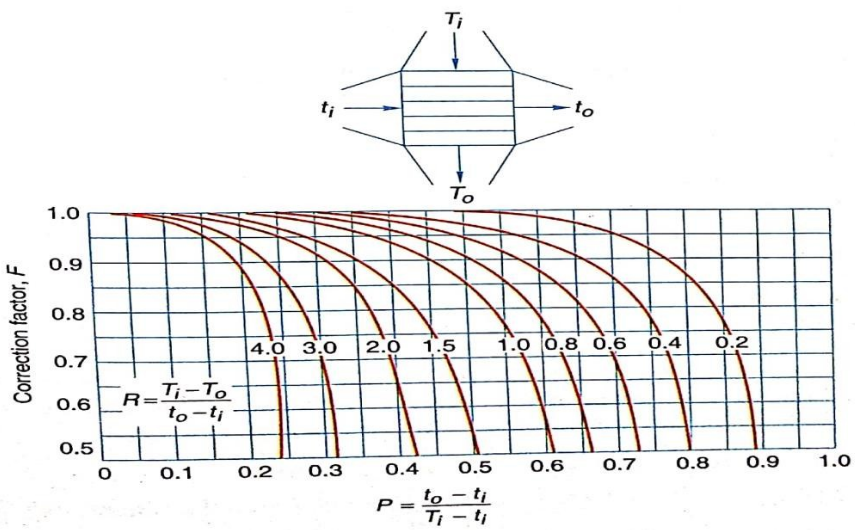


Fig. 12.12 Correction Factor Plot for Single Pass Cross Flow Heat Exchanger, One Fluid Mixed and the other Unmixed

Example 12.7

Saturated steam at 100°C is condensing on the shell side of a shell-and-tube heat exchanger. The cooling water enters the tubes at 30°C and leaves at 70°C. Calculate the effective log mean temperature difference if the arrangement is (*i*) counter flow, (*ii*) parallel flow and (*iii*) cross flow.

Solution

(*i*) *Counter flow*

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\text{In } (\Delta T_1 / \Delta T_2)} = \frac{(100 - 70) - (100 - 30)}{\text{In } (70/30)}$$

$$= \frac{30 - 70}{\text{In } (3/7)} = 47.2°\text{C}$$

(*ii*) *Parallel flow*

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\text{In } (\Delta T_1 / \Delta T_2)} = \frac{(100 - 30) - (100 - 70)}{\text{In } (70/30)}$$

$$= \frac{70 - 30}{\text{In } (7/3)} = 47.2°\text{C}$$

(*iii*) *Cross flow*

Referring to Fig. 12.12 for a single pass cross flow exchanger, one fluid mixed and the other unmixed, the value of the correction factor, F can be read off for the following values of P and R:

$$R = \frac{T_i - T_0}{t_0 - t_i} = \frac{100 - 100}{70 - 30} = 0$$

$$P = \frac{t_0 - t_i}{T_i - t_i} = \frac{70 - 30}{100 - 30} = \frac{4}{7} = 0.571$$

We observe that $F = 1.0$

$$\therefore \qquad F\Delta T_{lm} = 47.2°\text{C}$$

Thus we see that when one of the fluids in a heat exchanger, changes phase, (at constant temperature), it is immaterial whether we have parallel flow, counter flow or cross flow arrangements. The rate of heat transfer in all these modes will remain the same.

Example 12.8

In a food processing plant water is to be cooled from 18°C to 6.5°C by using brine solution entering at an inlet temperature of –1.1°C and leaving at 2.9°C. What area is required when using a shell-and-tube heat exchanger with the water making one shell pass and the brine making two tube passes? Assume an average overall heat transfer coefficient of 850 W/m^{2}K and a design heat load of 6000 W.

Also calc. for 2 shell & 4 tube pass.

Solution

The inlet and outlet temperatures of the tube and shell fluids are:

$$\text{Shell side}: T_i = 18^\circ\text{C}, \quad T_o = 6.5^\circ\text{C} \qquad (\text{Note: } T_o = \text{outlet temperature})$$

$$\text{Tube side}: t_i = -1.1^\circ\text{C}, \quad t_o = 2.9^\circ\text{C}$$

The LMTD for a counterflow arrangement would be given by Eqns. (12.20) and (12.22)

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{(18 - 2.9) - (6.5 + 1.1)}{\ln[(18 - 2.9)/(6.5 + 1.1)]}$$

$$= \frac{15.1 - 7.6}{\ln\left(\dfrac{15.1}{7.6}\right)} = \frac{7.5}{0.6865} = 10.92°\text{C}$$

The parameter P and R are evaluated as

$$P = \frac{t_o - t_i}{T_i - t_i} = \frac{2.9 + 1.1}{18 + 1.1} = \frac{4.0}{19.1} = 0.209$$

$$R = \frac{T_i - T_o}{t_o - t_i} = \frac{18 - 6.5}{2.9 + 1.1} = \frac{11.5}{4.0} = 2.875$$

Then the correction factor, from Fig. 12.9, for the above values of P and R is 0.97. The required heat transfer area is determined from Eqn. (12.24).

$$A = \frac{Q}{U(F\Delta T_{lm})} = \frac{6000}{(850)(0.97)(10.92)}$$

$$= 0.67 \text{ m}^2.$$

= 0.69

Example 12.9

Repeat Example 12.8 for two shell passes and four tube passes.

Solution

The values of R and P are the same as in the last example, but Fig. 12.10 must now be used to obtain F, which is ≈ 0.985.

$$\therefore \quad A = \frac{6000}{(850)(0.985)(10.92)}$$

$$= 0.656 \text{ m}^2.$$

12.6 EFFECTIVENESS—NTU METHOD OF HEAT EXCHANGER ANALYSIS

In the thermal analysis of various type of heat exchangers by the LMTD method, an equation of the type Eqn. (12.24) has been used. This equation is pretty simple and can be used in the design of heat exchangers when all the terminal temperatures are known or are easily determined. The difficulty arises if the temperatures of the fluids leaving the exchanger are not known. This type of situation is encountered in the selection of a heat exchanger or when the exchanger is to be run at off design conditions. Although the outlet temperature and heat flow rates can still be found with the help of the charts described earlier yet it would be possible only through a tedious trial and error procedure. In such cases, it is preferable to utilise an altogether different method known as the *Effectiveness–NTU* method.

The effectiveness method is based on the effectiveness of a heat exchanger in transferring a given amount of heat. To obtain an expression for the rate of heat transfer without involving any of the outlet temperatures let us first introduce the term effectiveness, $\in$, as

$$\text{Effectiveness} = \frac{\text{Actual heat transfer}}{\text{Maximum possible heat transfer}}$$

or

$$\in = \frac{Q}{Q_{max}} \tag{12.25}$$

The actual rate of heat transfer, Q, can be determined by either Eqns. (12.9) or (12.10). Q_{max} is the rate of heat transfer that a counterflow heat exchanger of infinite area would transfer with given inlet temperatures, flow rates and specific heats. Also we recognise that the maximum possible heat transfer would be obtained if one of the fluids was to undergo a temperature change equal to the maximum temperature difference present in the exchanger. We consider two distinct cases to illustrate this point.

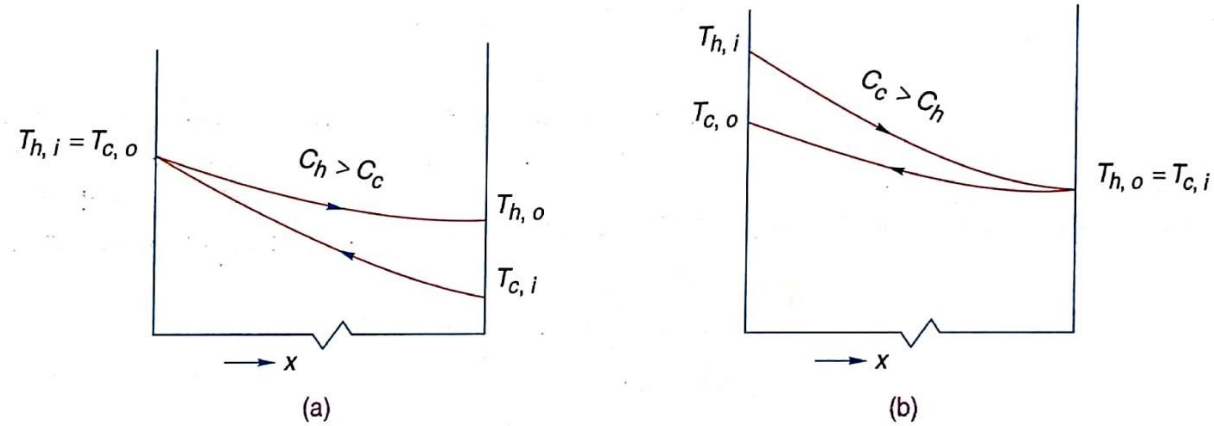


Fig. 12.13 Temperature Distribution in a Counter Flow Heat Exchanger of Infinitely Large Area

(*i*) $C_h > C_c$

For this type of exchanger, with no external heat losses, the outlet temperature of the cold fluid will equal the inlet temperature of the hot fluid (since the area available for heat transfer is infinite). The temperature distributions in the fluids are shown in Fig. 12.13 (*a*). The maximum rate of heat transfer is then given by

$$Q_{max} = C_c (T_{c,o} - T_{c,i})$$

But $$T_{c,o} = T_{h,i}$$

∴ $$Q_{max} = C_c (T_{h,i} - T_{c,i}) \quad (12.26)$$

Also $$Q = C_c (T_{c,o} - T_{c,i}) \quad (12.10)$$

(*ii*) $C_h < C_c$

In this case the outlet temperature of the hot fluid would equal the inlet temperature of the cold fluid, as shown in Fig. 12.13 (*b*). So

$$Q_{max} = C_h (T_{h,i} - T_{h,o})$$

$$T_{h,o} = T_{c,i}$$

But

$$Q_{max} = C_h (T_{h,i} - T_{c,i}) \quad (12.27)$$

∴

$$Q = C_h (T_{h,i} - T_{h,o}) \quad (12.9)$$

Also

Looking at Eqns. (12.26) and (12.27) we may write the general expression

$$Q_{max} = C_{min} (T_{h,i} - T_{c,i}) \quad (12.28)$$

where C_{min} is the smaller of C_c and C_h. Using Eqn. (12.25) as the definition of effectiveness, it follows that:

$$\in = \frac{C_h (T_{h,i} - T_{h,o})}{C_{min} (T_{h,i} - T_{c,i})} \quad (12.29)$$

or

$$\in = \frac{C_c (T_{c,o} - T_{c,i})}{C_{min} (T_{h,i} - T_{c,i})} \quad (12.30)$$

Once the effectiveness for a heat exchanger is known, its actual rate of heat transfer can be determined by

$$Q = \in C_{min} (T_{h,i} - T_{c,i}) \quad (12.31a)$$

$$Q = \in Q_{max} \quad (12.31b)$$

Equation (12.31) is very significant because it expresses the actual rate of heat transfer by a heat exchanger in terms of its effectiveness, C_{min} and the difference between the inlet temperatures of the two fluids. It *does not* refer to the outlet fluid temperatures and can replace the LMTD analysis effectively.

As will be shown in the following subsections, effectiveness for any heat exchanger can be expressed

$$\in = \in\left(\frac{UA}{C_{\min}}, \frac{C_{\min}}{C_{\max}}\right) \tag{12.32}$$

where $\frac{C_{\min}}{C_{\max}} = \frac{C_c}{C_h}$ or $\frac{C_h}{C_c}$ (depending upon their relative magnitudes).

The group $\frac{UA}{C_{\min}}$ is called the *number of transfer units, NTU.*

Thus
$$NTU = \frac{UA}{C_{\min}} \tag{12.33}$$

NTU is a dimensionless parameter. It is a measure of the heat transfer size of the exchanger. The larger the value of *NTU*, the closer the heat exchanger reaches its thermodynamic limit of operation.

12.6.1 Effectiveness for a Parallel-Flow Heat Exchanger

Let us now determine the specific form of the effectiveness. *NTU* relation for a parallel flow heat exchanger first. Assuming $C_{\min} = C_c$, $\in$ from Eqn. (12.30) is

$$\in = \frac{T_{c,o} - T_{c,i}}{T_{h,i} - T_{c,i}} \tag{12.34}$$

From Eqns. (12.9) and (12.10) we get

$$\frac{C_{\min}}{C_{\max}} = \frac{\dot{m}_c\, c_c}{\dot{m}_h\, c_h} = \frac{T_{h,i} - T_{h,o}}{T_{c,o} - T_{c,i}}$$

Rearranging Eqn. (12.16) in the form

$$\ln\left(\frac{\Delta T_2}{\Delta T_1}\right) = \ln\left(\frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}}\right) = \frac{-UA}{C_{\min}}\left(1 + \frac{C_{\min}}{C_{\max}}\right) \quad (12$$

or from Eqn. (12.33)

$$\frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}} = exp\left[-NTU\left(1 + \frac{C_{\min}}{C_{\max}}\right)\right] \quad (1$$

The left side of Eqn. (12.36) can be rearranged as

$$\frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}} = \frac{T_{h,o} - T_{c,i} + T_{c,i} - T_{c,o}}{T_{h,i} - T_{c,i}}$$

which on substitution of the value of $T_{h,o}$ from Eqn. (12.35) becomes

$$\frac{T_{h,o} - T_{c,o}}{T_{h,i} - T_{c,i}} = \frac{(T_{h,i} - T_{c,i}) - \dfrac{C_{\min}}{C_{\max}}(T_{c,o} - T_{c,i}) - (T_{c,o} - T_{c,i})}{T_{h,i} - T_{c,i}}$$

$$= 1 - \left(\frac{C_{\min}}{C_{\max}}\right)\in - \in = 1 - \in\left(1 + \frac{C_{\min}}{C_{\max}}\right)]$$

Going back to Eqn. (12.36) we get

$$1 - \in\left(1 + \frac{C_{\min}}{C_{\max}}\right) = exp\left[-NTU\left(1 + \frac{C_{\min}}{C_{\max}}\right)\right]$$

or

$$\in = \frac{1 - exp\,\{-NTU\,\{1 + (C_{\min}/C_{\max})\}}{[1 + (C_{\min}/C_{\max})\}} \quad (1$$

Notice that the expression for $\in$ contains U, A and the heat capacities only. Also had we started $C_{\min} = C_h$, we would have obtained the same expression for $\in$.

12.6.2 Effectiveness for a Counter Flow Heat Exchanger and other Configurations

From an analysis like that made in the preceding section, the following relation for effectiveness in a *counter flow heat exchanger* can be obtained

$$\in = \frac{1 - exp\,\{-NTU\,[1 - (C_{min}/C_{max})]\}}{1 - (C_{min}/C_{max})\,\{exp - NTU\,[1 - (C_{min}/C_{max})\}} \qquad (12.38)$$

Flow geometry	Relation
Double pipe:	
Parallel Flow	$\in = \frac{1 - exp\left[-N\,(1 + C)\right]}{1 + C}$
Counter Flow	$\in = \frac{1 - exp\left[-N\,(1 - C)\right]}{1 - C\,exp\left[-N\,(1 - C)\right]}$
Cross Flow:	
Both Fluids unmixed	$\in = 1 - exp\left\{\frac{C}{n}\left[exp\,(-NCn) - 1\right]\right\}$, where $n = N^{-0.22}$
Both Fluids mixed	$\in = \left[\frac{1}{1 - exp(-N)} + \frac{C}{1 - exp(-NC)} - \frac{1}{N}\right]^{-1}$
C_{max} mixed, C_{min} unmixed	$\in = (1/C)\left\{1 - exp\left[C\left(1 - e^{-N}\right)\right]\right\}$
C_{max} unmixed, C_{min} mixed	$\in = 1 - exp\left\{1/C\left[1 - exp\,(-NC)\right]\right\}$
Shell-and-Tube:	
One shell pass, 2, 4, 6 tube passes	$\in_1 = 2\left\{1 + C + (1 + C^2)^{1/2}\frac{1 + exp\left[-N\,(1 + C^2)^{1/2}\right]}{1 - exp\left[-N\,(1 + C^2)^{1/2}\right]}\right\}^{-1}$
Two shell pass, any multiple of 4 tubes	$\in_2 = \left[\left(\frac{1 - \in_1 C}{1 - \in_1}\right)^2 - 1\right]\left[\left(\frac{1 - \in_1 C}{1 - \in_1}\right)^2 - C\right]^{-1}$

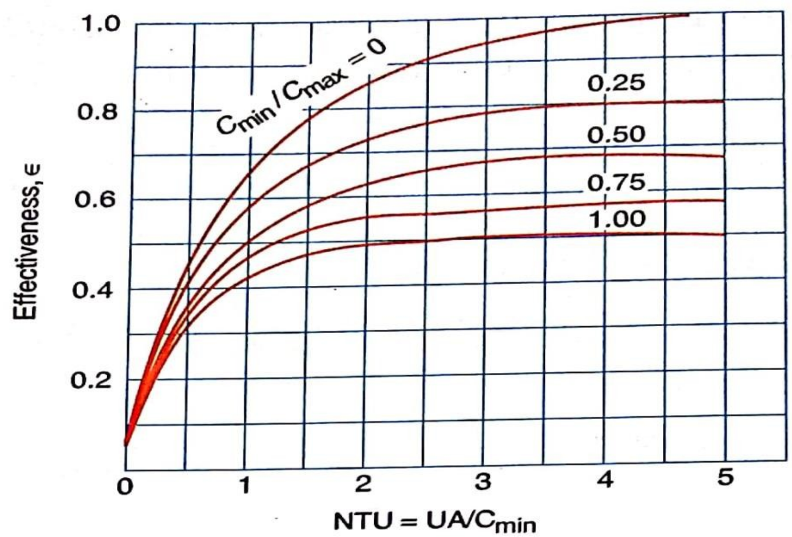


Fig. 12.14 Effectiveness for Parallel Flow Heat Exchanger

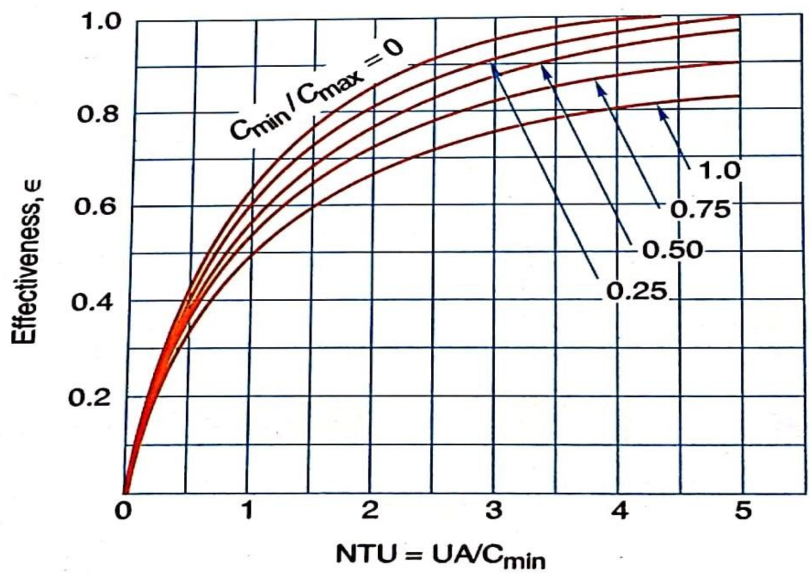


Fig. 12.15 Effectiveness for Counter Flow Heat Exchanger

C_h 232.63

Example 12.13

Water enters a counter flow, double pipe heat exchanger at 15°C, flowing at the rate of 1300 kg/h. It is heated by oil $(C_p = 2000\ \text{J/kgK})$ flowing at the rate of 550 kg/h from the inlet temperature of 94°C. For an area of 1 m^2 and an overall heat transfer coefficient of $1075\ \text{W/m}^2\text{K}$, determine the total heat transfer and the outlet temperatures of water and oil.

Solution

Taking the specific heat of water as 4186 J/kg K the heat capacity rates are

water:
$$C_c = \dot{m}_c C_c = \frac{(1300)\,(4186)}{(3600)} = 1511.61\ \text{W/K}$$

oil:
$$C_h = \dot{m}_h C_h = \frac{(500)(2000)}{(3600)} = 305.55\ \text{W/K}$$

in which case

$$C_{min} = C_h = 305.55\ \text{W/K}$$

and

$$\frac{C_{min}}{C_{max}} = \frac{305.55}{1511.61} = 0.2$$

also $$NTU = \frac{UA}{C_{min}} = \frac{(1075)(1)}{305.55} = 3.52$$

From Fig. 12.15 the heat exchanger effectiveness is

$$\in \simeq 0.94$$

$$\therefore \quad Q_{max} = C_{min}\,(T_{h,i} - T_{c,i}) = (305.55)(94 - 15) = 24138.5 \text{ W}$$

$\therefore$ Actual heat transfer $Q = \in Q_{max} = 22690.2$ W

Then by energy balance,

Outlet temperature of water $T_{c,o} = \dfrac{Q}{C_c} + T_{c,i}$

$$= \frac{22690}{1511.61} + 15 = 30°\text{C}$$

Outlet temperature of oil, $T_{h,o} = T_{h,i} - \dfrac{Q}{C_h}$

$$= 94 - \frac{22690}{305.55} = 19.74°\text{C}.$$

Water enters a cross flow heat exchanger (both fluids ummixed) at 5°C and flows at the rate of 4600 kg/h to cool 4000 kg/h of air that is initially at 40°C. Assume the U value to be 150 $W/m^2 K$. For an exchanger surface area of 25 m^2, calculate the exit temperature of air and water.

Solution

Taking the specific heats of water and air to be constant at 4180 J/kgK and 1010 J/kgK respectively, we have

Air:
$$\dot{m}_h C_h = \frac{(4000)(1010)}{3600} = 1122.22 \text{ W/K}$$

Water:
$$\dot{m}_c C_c = \frac{(4600)(4180)}{3600} = 5341.11 \text{ W/K}$$

in which case

$$C_{min} = C_h = 1122.22 \text{ W/K}$$

and
$$\frac{C_{\min}}{C_{\max}} = \frac{1122.22}{5341.11} = 0.21$$

$$NTU = \frac{UA}{C_{\min}} = \frac{(150)(25)}{1122.22} = 3.34$$

From Fig. 12.18 the effectiveness is then

$$\in = 0.92$$

The heat transfer rate, Q, is given by

$$Q = \in C_{\min} (T_{h,i} - T_{c,i})$$

$$= (0.92)\ (1122.22)\ (40 - 5) = 36135.5 \text{ W}$$

Then by energy balance,

outlet temperature of water, $T_{c,o} = \frac{Q}{C_c} + T_{c,i} = \frac{(36135.5)}{(5341.11)} + 5$

$= 11.8°C$

outlet temperature of air, $T_{h,o} = T_{h,i} - \frac{Q}{C_h} = 40 - \frac{36135.5}{1122.22}$

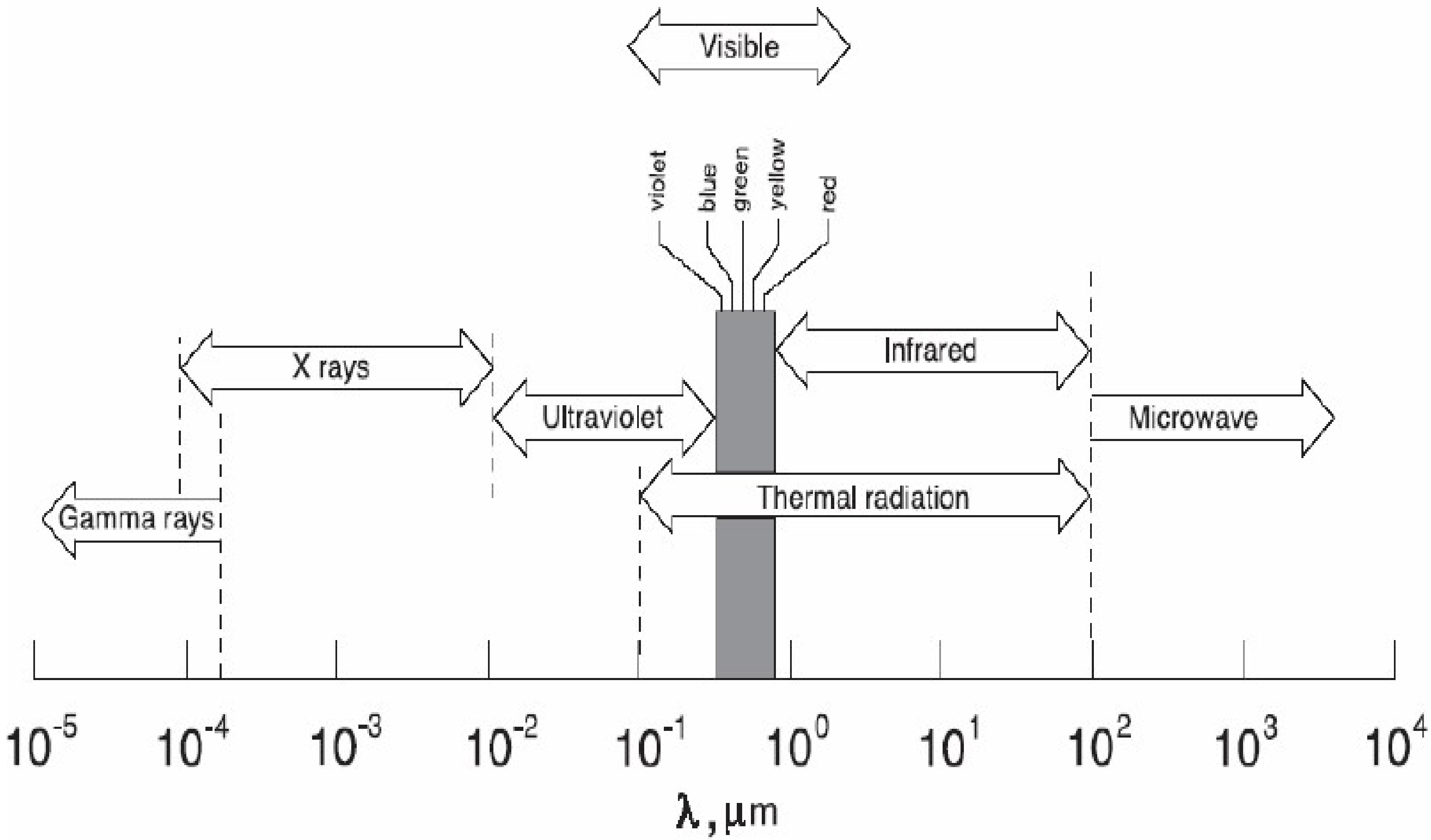
$= 7.8°C$

V - UNIT

Radiation

- **Thermal radiation** is an electromagnetic phenomenon generated by the thermal motion of particles in matter.
- All matter with a temperature greater than absolute zero emits thermal radiation.
- All bodies emit radiation to their surroundings through electromagnetic waves due to the conversion of the internal energy of the body into radiation.
- Particle motion results in the charge acceleration which produces electromagnetic radiation.

- Since electromagnetic waves can also travel through a vacuum hence, in contrast to the conduction and convection heat transfer, it can take place through a perfect vacuum.
- Thus, when no medium is present, radiation becomes the only mode of heat transfer.
- Common examples are the solar radiation reaching the earth and the heat dissipation from the filament of an incandescent lamp.
- Thus heat is transferred between two bodies over a great distance.



Electromagnetic spectrum.

Type of rays	Wavelength λ, μm
Cosmic rays	up to 4×10^{-7}
Gamma rays	4×10^{-7} to 1×10^{-4}
X-rays	1×10^{-5} to 2×10^{-2}
Ultraviolet rays	1×10^{-2} to 0.38
Visible (light)	0.38–0.78
Infrared rays	
Near	0.78–25
Far	25–1000
Thermal radiation	0.1–1000
Radar, television and radio	1×10^{3} to 2×10^{10}

Spectrum of electromagnetic radiation

- Waves falling in the range of 0.1 to 100μm wave length are called thermal radiation
- According to the quantum theory, the thermal radiation propagates in the form of discrete quanta, each quantum having an energy of

$$E=h\nu$$

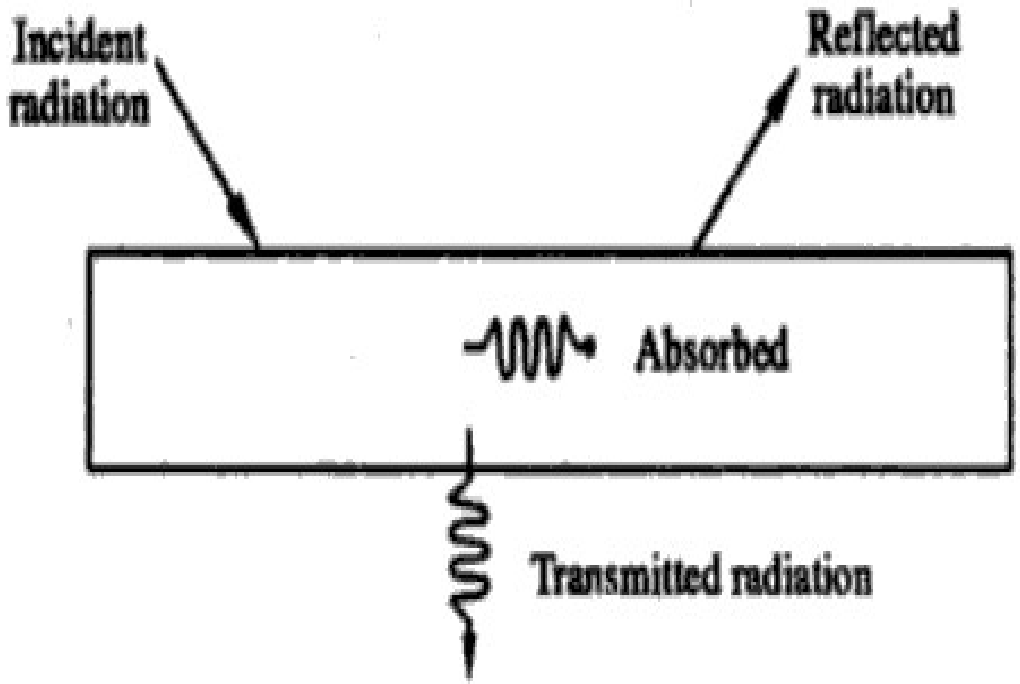
Where h = Planck's constant = $6.625*10^{-34}$ J-s

ν = Frequency of quantum

Reflection, Absorption, and Transmission of Radiation

- When radiation falls on a body, a part of it may be absorbed, a part may be reflected and the remaining may pass through the body.
- The fraction of the incident radiation absorbed by the body is transformed into heat.

When radiation energy is incident on a body, it is partially reflected, partially transmitted and partially absorbed as shown in Fig. 7.3. The *reflectivity* ρ is defined as the fraction of the incident radiation reflected from the surface of the body.



Reflection, transmission and absorption of radiation.

$$Q = Q_A + Q_R + Q_T$$

Dividing both sides of the equation by Q, we get

$$\frac{Q_A}{Q} + \frac{Q_R}{Q} + \frac{Q_T}{Q} = 1$$

The first fraction in the equation is known as *absorptivity* α, second is *reflectivity* ρ, and the third fraction is *transmissivity* τ. Hence,

$$\alpha + \rho + \tau = 1$$

- The reflectivity is defined as the fraction of incident radiation reflected from the surface of the body.
- The transmissivity is defined as the fraction of the incident radiation transmitted through the body .
- The absorptivity is defined as the fraction of incident radiation absorbed by the body.
- Bodies which do not transmit radiation are called opaque.

If the transmissivity τ of a body is equal to one, the absorptivity and reflectivity are equal to zero and whole of the incident radiation would pass through the body. Such a body is termed as absolutely *transparent or diathermanous*. The only substance found to be perfectly diathermanous is crystalline pieces of rock salt. Air has nearly zero absorptivity and reflectivity. However, polyatomic gases, such as carbon dioxide, methane, and water vapour are capable of absorbing heat radiation.

- A body with reflectivity of unity will reflect whole of the incident radiation and is termed as white body.

Concept of a Black body

If the entire incident radiation is absorbed by the body, the absorptivity $\alpha = 1$. Such a body is termed as a *blackbody*. Only a few surfaces, such as carbon black, platinum black, and gold black, approach the absorption capability of a blackbody. It is to be noted that the blackbody derives its name from the observation that surface appearing black to the eye is normally good absorber of incident visible light.

- No actual body is perfectly black, the concept of a black body is an idealization with which the radiation characteristics of real bodies can be conveniently compared.
- Real bodies do not emit as much as energy as black body and hence their emissivity is less than one.
- A black body plays a role in thermal radiation similar to the idealized Carnot cycle in thermodynamics with which real cycles are compared.
- A black body is regarded as a perfect absorber of incident radiation.

- The total radiation emitted by a black body is a function of temperature.
- The emissivity of a substance is a measure of its ability to emit radiation in comparison with a black body.
- A black body is a perfect emitter.
- Intensity of radiation is defined as the radiation emitted in any direction.
- The radiation intensity of a surface is defined as the rate of heat flux emitted by it per unit area.

Laws of Radiation

1. Planck's Law:

- Electromagnetic radiation consists of flow of quanta or particles and the energy content (E) of each quantum is proportional to the frequency.
- It is given by the following equation:
- E = hv

  Where, E = Energy content

  h = Planck's constant = 6.625 x 10^{-34} J.s

  v = Frequency
- It is clear that greater the frequency, shorter the wavelength and greater is the energy content of the quantum. In other words, shorter the wavelength greater is the energy of the quantum. Therefore, quanta of ultraviolet light are more energetic than are quanta of red light.

2. Kirchoff's Law:

- Kirchoffs law states that the absorptivity (a) of a substance for radiation of a specific wavelength is equal to its emissivity for the same wavelength and is given by the following equation:

$$a(\lambda) = e(\lambda)$$

- Any grey object (other than a perfect black body) which receives radiation, disposes off a part of it in reflection and transmission.
- The absorptivity, reflectivity and transmissivity are each less than or equal to unity.

Kirchhoff's Law

The law states that at any temperature the ratio of emissive power E to the absorptivity α is a constant for all bodies and equals the emissive power of a blackbody at the same temperature, i.e.,

$$\frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2} = \frac{E_3}{\alpha_3} = \cdots = E_b = f(T)$$

Since the ratio of the emissive power of a gray body to that of a blackbody at the same temperature is defined as emissivity, hence

$$\frac{E_1}{E_b} = \alpha_1 = \varepsilon_1; \frac{E_2}{E_b} = \alpha_2 = \varepsilon_2$$

$$\frac{E_1}{E_b} = \alpha_1 = \varepsilon_1; \frac{E_2}{E_b} = \alpha_2 = \varepsilon_2$$

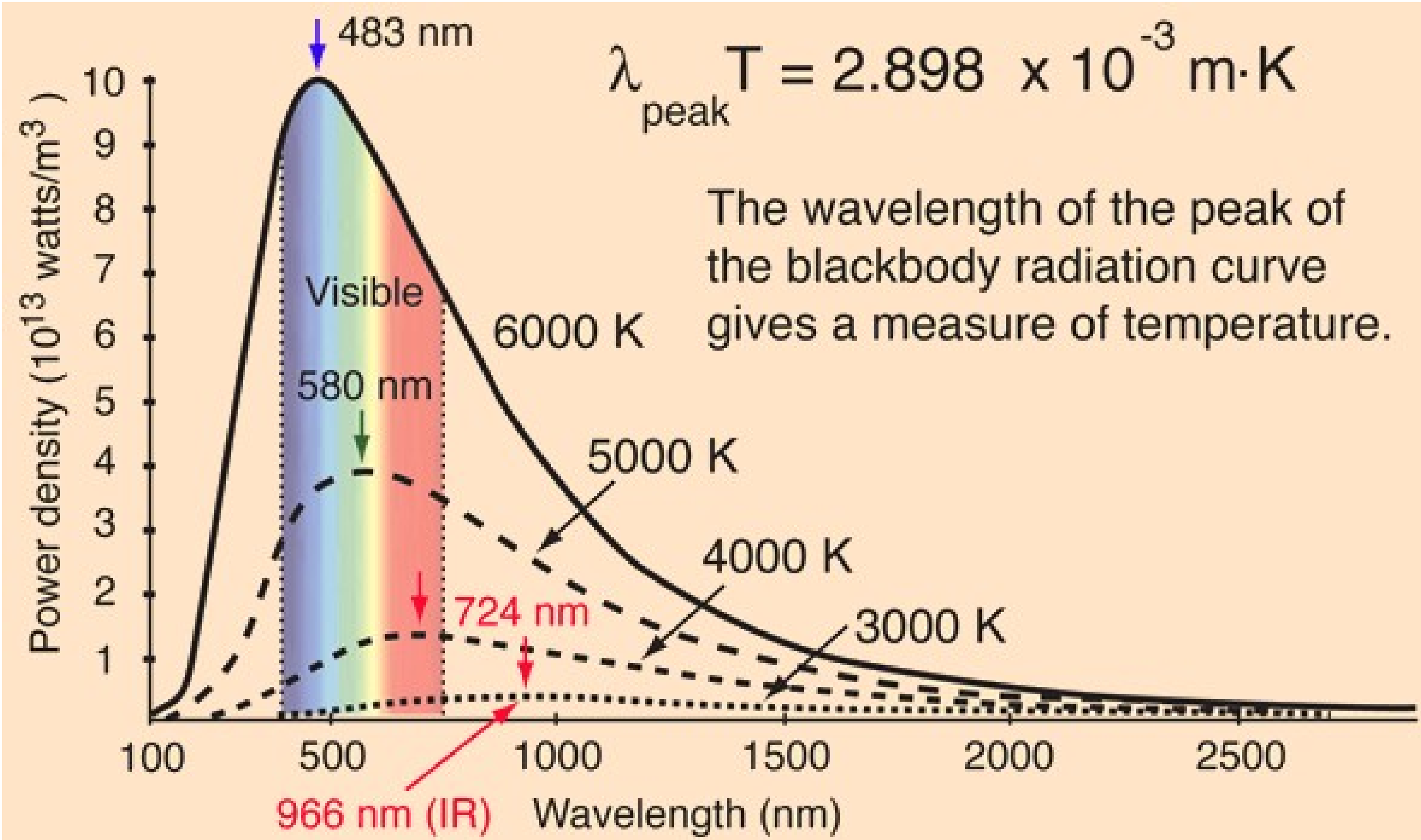
For monochromatic radiation, the law states that the ratio of the emissive power at a certain wavelength to the absorptivity at the same wavelength is the same for all bodies and is a function of wavelength and temperature, i.e.,

$$\frac{(E_\lambda)_1}{(\alpha_\lambda)_1} = \frac{(E_\lambda)_2}{(\alpha_\lambda)_2} = \frac{(E_\lambda)_2}{(\alpha_\lambda)_2} = \cdots = E_b = f(\lambda, T)$$

- Monochromatic radiations are such radiations which are characterized by a single frequency.
- In practice, radiation of a very small range of frequencies which can be described by stating a single frequency.

Wien's Displacement Law

- When the temperature of a blackbody radiator increases, the overall radiated energy increases and the peak of the radiation curve moves to shorter wavelengths.
- When the maximum is evaluated from the Planck radiation formula, the product of the peak wavelength and the temperature is found to be a constant.



$$\lambda_{peak}T = 2.898x10^{-3}m \cdot K$$

Wien established a relationship between the temperature of a black body and the wavelength at which the maximum value of monochromatic emissive power occurs. A peak monochromatic emissive power occurs at a particular wavelength. **Wien's displacement law** *states that the product of* λ_{max} *and T is constant, i.e.,*

- Wavelength (λ_{max}) of maximum intensity of emission (μ) = b/T

Where,

- λ_{max} is the wavelength at which maximum radiation is emitted. It decreases as the temperature increases.

  b is constant = 2897

  T is the temperature of the surface in Kelvin

Hence, λ_{max} (μ) = 2897 T^{-1}

$$\text{Maximum wavelength } (\lambda_{max}) \text{ for sun} = \frac{2897\ (\mu^{o}K)}{6000\ (^{o}K)} = 0.483\ \mu \text{ or } 0.5\mu$$

$$\text{Maximum wavelength } (\lambda_{max}) \text{ for earth} = \frac{2897\ (\mu^{o}K)}{300\ (^{o}K)} = 9.66\ \mu \text{ or } 10.0\mu$$

- The temperature of the sun is 6000 °K for which the value of maximum wave length is 0.5μ, and that of the earth the average temperature is 300 °K for which the value of maximum wavelength is 10μ.
- Out of the total energy emitted by sun, 7 per cent is with wavelength less than 0.4μ, 44 per cent is with a wavelength ranging from 0.4 – 0.7μ and 49 per cent is having wavelength greater than 0.7μ.

Stefan-Boltzman's Law:

This law states that the intensity of radiation emitted by a radiating body is proportional to the fourth power of the absolute temperature of that body.

Radiation Heat Transfer, $Q = \varepsilon \sigma T^4$

Where,

σ = Stefan-Boltzman's constant =6.25*10-34 Js

ε = Emissivity of a body ($0 \leq s \geq 1.0$)

T = Absolute temperature of the surface in °K.

LAMBERT'S COSINE LAW

The law states that the *total emissive power* E_θ from *a radiating plane surface in any direction is directly proportional to the cosine of the angle of emission*. The angle of emission θ is the angle subtended by the normal to the radiating surface and the direction vector of emission of the receiving surface. If E_n be the total emissive power of the radiating surface in the direction of its normal, then

$$E_\theta = E_n \cos\theta$$

The above equation is true only for diffuse radiation surface. The radiation emanating from a point on a surface is termed diffused if the intensity, I, is constant. This law is also known as *Lambert's law of diffuse radiation.*

Radiation Shape Factor

- Radiation shape factor is defined as the fraction of radiant energy that is diffused from one surface element and strikes the other surface directly with no intervening reflections.
- It is also called view factor or configuration factor.
- If A11 is the total area of radiating surface of body-1 having shape factor F12F12 w.r.t. receiver body-2 then the total radiant energy leaving surface-1 and directly intercepted by surface-2 is =A1F12

Shape factor of a radiant body depends on the

1. Geometrical dimensions i.e. surface area
2. Configuration of radiating surface with respect to receiver &
3. Inter-spatial distance of radiant body with respect to receiver.

- The shape factor of a radiating body is inversely proportional to its surface area emitting radiant energy i.e.

  Shape factor a 1/Surface area of emitter
- The shape factor of a radiating body is directly proportional to the surface area of receiving body i.e.

  Shape factor a Surface area of receiver

- The shape factor of a radiating body is inversely proportional to inter-spatial distance between emitter and receiver bodies i.e.

  Shape factor a 1 / Inter-spatial distance

- For steady state condition of radiation heattransfer,

  Rate of radiant energy lost by body-1 = rate ofradiant energy received by body-2
- $A_1F_{12}=A_2F_{21}$